REVIEW OF RESEARCH
ISSN: 2249-894X
IMPACT FACTOR : 5.7631(UIF)
VOLUME - 8 | ISSUE - 11 | AUGUST - 2019

THE BIOLOGY OF PRIMES: A NEW APPROACH ON PRIME NUMBERS

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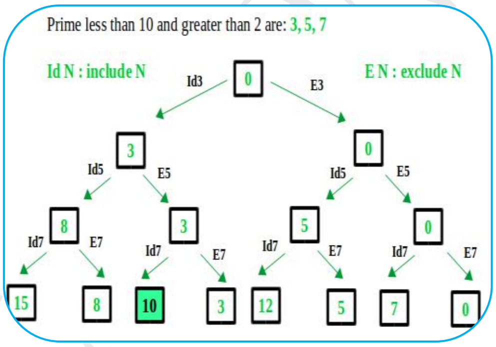
ABSTRACT

This essay proposes to demonstrate how the use of numeral properties may facilitate the understanding of prime numbers, allowing both the obtaining, and the verification of these numbers, dismissing, completely, the use of Riemann's hypothesis (permitting the access to this kind of study, to undergraduate students, even at the fundamental grades), and showing itself as a simple, viable, and yet, definitive solution to the problem of finding prime numbers.



KEYWORDS: *Prime numbers; Riemann hypothesis; Number families, genus and species.*

INTRODUCTION

The search for a formula, or test, capable of determining if a number is or not a prime, and furthermore, a formula that can produce prime numbers securely, is being held by mathematicians throughout the centuries, without success, and has become even more fierce with the advent of internet and the use of these numbers in various areas, such as cryptography (specially asymmetric).

It so happens that the methods, existing up to this date, besides being extremely complex, resent the precision needed for their functionality, and present themselves as all but accessible to students at the fundamental grades, which in turn, contributes to making math appear tedious and unattractive to students in general.

It's necessary, therefore, to search for a new formula, simple enough, as to be taught to fundamental grade infants, and that, simultaneously, be precise in all its results, so as to dismiss other tests, or calculations.

Evidently, one such formula would have to limit itself to the common concepts and knowledge of this stage of learning, and should, therefore, avoid the use of intricate and difficult mathematical models, however correct.

To do so, a pioneering approach is suggested, starting with the analysis of the numerical properties that composes each number to, then, learning how to generate prime numbers, as well as, to the reverse, verifying if a given number is prime or not.

These numerical properties, will allow us to classify the number into families, genus and species, which, then, will serve to the purpose that is intended in this approach.

It must be warned, however, that, howbeit the modular arithmetic, eventually, could be used to solve the problems approached here, we have purposely, preferred the use of linear equations, as they seem simpler and present themselves closer to the goal laid before, of maintaining only the use of formulations, capable of been taught to fundamental grade students.

The properties analyzed here, could, perhaps, be presented in relation to all the basic operations, although only two would be enough to obtain prime numbers and verify its primality, which is the reason why, in this essay, only the needed properties shall be used.

The need for such approach, is evidenced by the fact that the methods used today, all resent being only approximations, without the necessary accuracy, besides being too complex, making it impossible to teach them at the fundamental or basic levels of schooling.

One must search for more accessible formulas, less complex (if not, totally incomprehensible), and that allow the study of this particular area of math, even to high school alumni, or fundamental grade students.

The solution to the problem today, is been seeked, very intensely, in the handling of the known Riemann's zeta function, whose proof, however, remains a challenge.

In part, this difficulty in finding the formula that would solve the prime problem for good, is precisely the fact that virtually all those who have tried, have insisted in approaching the problem as if all prime numbers were but of a single kind.

But as it will be shown in this essay, this kind of mistake is the same as trying to describe all living beings, as if all of them were of the same kind. Prime numbers, just as living beings, present themselves in groups, such as families, genus and species.

Understanding this very important aspect, will give us the tools needed to solve the problem once and for all.

Unfortunately, the search for the proof of the Riemann's zeta function, escalated so much, that all other manners of facing the prime function problem have been, somewhat, abandoned or dismissed.

Note that the Riemann formula, certainly, is too complex to be taught to students at the fundamental level, thence the need to search for a parallel and more palatable solution to the amount of knowledge acquired by students at this level of learning, just when such learning is administered.

The present article has the ambition of, precisely, bring back the debate on manners to solve the prime problem, through a different approach from that formulated, initially, by Gauss, Euler and Riemann[1] and, at the same time, show that even through functions, simple enough to be taught to students at the fundamental level, it is possible to tackle the subject and find solutions.

1 – PREVIOUS ATTEMPTS

As has been mentioned, most of the work done today about prime numbers, revolves around the Riemann hypothesis, but it must be remembered that it has not always been this way.

In fact, the study of perfect numbers[2] done by Euclides, already would show that there could be prime numbers of similar properties, since many primes would not fit in the category of perfect number generators.

Also worthy to be noted was the sieve of Eratosthenes, who designed a method of finding primes within certain limits, which basically consisted of making *"a list of all the integers less than or equal to n (and greater than one), [then striking] out the multiples of all primes less than or equal to the square root of n"*[3] which would leave only the prime numbers.

But it was Mersenne who, working with the perfect numbers, conjectured a formula to find certain kinds of prime numbers.

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1 The formulas searched by them, in fact, only determined the amount of numbers in a determined interval and not, properly, which numbers would these be. The Riemann's non trivial zeros would indicate, precisely, the prime number in the critical line.

2 A perfect number is a natural number who is equal to the sum of its divisors (except for itself), as shown by SILVA, Marcos Noé Pedro da. "Mersenne, Números Primos e Números Perfeitos "; Brasil Escola. Available at <https://brasilescola.uol.com.br/matematica/mersenne-numeros-primos-numeros-perfeitos.htm>. Access on 04/16/2019.

3 Available at: https://primes.utm.edu/glossary/page.php?sort=SieveOfEratosthenes . Access on 04/22/2019.

In fact, the Mersenne prime numbers (today up to 51), represent but a set of prime numbers, which, by the very small amount of cases, already show that there are different properties for primes, which could be explored for the solution.

A Mersenne prime number is a prime number which can be described as M_n =(2^n)-1. However, due to the great amount of digits that such numbers require it is only by the use of super computers that they can be calculated and, so far, only 51 primes have been found to fit such a formula, although the search continues for other numbers.

The greatest Mersenne prime number known so far (the 51th) is 2^82,589,933 -1 which has 24,862,048 decimal digits. and was discovered at 2018[4], what clearly shows that it does not correspond to a formula that could give all prime numbers, but only a very few and of a specific kind.

The fact that it can only give a very specific kind of primes, already serve to demonstrate that primes can be described in groups.

Later, Fermat would take up Mersenne's work and show that there could be another set of primes (which he believed to be the ultimate solution to the prime problem[5]), that could be represented by the formula P = 2^(2^n) – 1.

This formula was believed to be the answer to the prime problem, until it was proven by Euler that for n=5, the Fermat formula resulted in a number divisible by 641.

In fact, up to this date, "*it is still unknown if there are other Fermat primes beyond the first initial 5 for great values of n*"[6].

It must be remembered, however, that Euclids, around 300 BC, had already stated that "*if a number written in the form (2^n)-1 is a prime, then this number is a perfect number*"[7].

And much later (1747), Euler demonstrated that all perfect even numbers are of the form *(2^n)-1*[8].

Euler would also be responsible for the formula p = n^2-n+41, which is good for finding all primes up to 41, but fails afterward.

After Euler, the search about primes began to focus, not so much on how to find or determine primes, but rather on counting how many primes would there be in a given interval.

The first to conjecture a prime number theorem was Legendre, who, in 1798, *"stated that if x is not greater than 1,000,000, then x/(ln(x) – 1.08366) is very close to π(x)"*[9], or, in other words, the amount of prime numbers in that interval would be close to that function.

The formula contained a constant, known as the Legendre constant, that would later be dismissed by Gauss, who estimated that *π(n)~ n/ln(n).*

A while later, Gauss himself, "refined his estimate to π(n)~ Li(n) where is the is the logarithmic integral"[10].

$$\mathrm{Li}(n) = \int_2^n \frac{dx}{\ln x}$$

As can be easily seen, even this function had a certain error, for it was a good approximation, but which needed to be solved.

4 As shown by https://www.mersenne.org/primes/ . Access on 08/27/2019.
5 As shown by ARAÚJO, Gabriel Lúcio de et alii , Primos de Fermat, Primos de Mersenne, N´umeros Perfeitos e O Fatorial, available at https://www.ime.unicamp.br/~ftorres/ENSINO/MONOGRAFIAS/Maicon_TN17M2.pdf . Access on 04/16/2019.
6 Idem.
7 In http://www.educ.fc.ul.pt/icm/icm98/icm12/Historia.htm Access on 04/17/2019.
8 idem
9 Available at https://www.britannica.com/science/prime-number-theorem . Access on 04/23/2019.
10 Available at Weisstein, Eric W. "Prime Number Theorem." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/PrimeNumberTheorem.html . Access on 04/23/2019.

Thus, analyzing this very approximation, Riemann stated his famous hypothesis, yet unproven, that

“ζ(z)=∑∞n=11nz=11z+12z+...ζ(z)=∑n=1∞1nz=11z+12z+...

where z=x+iyz=x+iy is a complex number, so ζ(z)ζ(z) is also complex”[11].

In general, it means that the sum of primes in a given interval can be determined by the non trivial zeros of the zeta function.

However, it has been showed that even this function is still an approximation, whose difference (error) tends to zero, as it gets closer to infinity.

“In particular the error term in the prime number theorem is closely related to the position of the zeros. For example, if β is the upper bound of the real parts of the zeros, then *π(x)-li(x)=0 (x^ β log x)”*[12].

In 1976, *“Schoenfeld, showed that the Riemann's hypothesis, implied that* $|\psi(x)-x| < \frac{1}{8\pi}\sqrt{x}\log^2(x)$, *for all x≥ 73.2, where ψ(x) is Chebyshev's second function”*[13].

As it has been stated, nowadays, the search for primes is almost limited to the search for the amount of prime numbers in a given interval, rather than to find the numbers themselves.

In this attempt, we shall show that returning the aim at the primes themselves, can give a much easier and yet better solution to the prime problem, without recurring to tools outside of the number theory field.

2 – NUMBER REDUCTION

All integers, greater than zero, can be reduced to only nine groups (which we have designated as base families), from which, it becomes easier and more advantageous to study them.

The importance of this classification and the reduction itself, resides in, first, identify the patterns of these numerals and verify that each family will have one respective property, that will facilitate the comprehension and study of the numbers they represent.

It's good to keep in mind that numbers are abstract concepts, represented by numerals, that can, in fact, be different from the more common representation, known as arabic. Indeed, there are other numerals known worldwide, such as romans, egyptians, etc.

In this essay, only the arabic numerals shall be used, for they are more widely known and have the advantage of being capable to represent infinite numbers.

CONCEPT: The reduction of a number is equal to the sum of the numerals that compose it, repeating the same adding operation, until it is reduced to only one algorithm (integer).

So, we can express the reduction, mathematically, as follows:

Let there be any number, such as, for example: 12345

Its reduction shall be obtained as follows: 1+2+3+4+5 = 15. Then: 1+5 = 6.

Therefore, the number 12345 belongs to the family base 6, or, alternatively: 12345 has reduction R6.

From this reduction, it can be seen that all number will fit in determined families, and these shall repeat periodically, in such way, that there will never be numbers (except zero) that will not fit in, at least, one of these families.

Hence, the families are, namely: R1;R2.R3...R9.

11. Available at https://www.quora.com/What-is-the-Riemann-Zeta-Function-and-what-are-its-purpose-and-uses . Access on 04/23/2019.
12. Available at https://en.wikipedia.org/wiki/Riemann_hypothesis#Riemann_zeta_function . Access on 04/23/2019.
13. Available at https://en.wikipedia.org/wiki/Riemann_hypothesis#Riemann_zeta_function . Access on 04/23/2019.

The first advantage of this type of operation, and mostly known as well, is the verification that a few of them will always be multiples of just one number.

In fact, the reduction R3; R6 and R9 will always be multiples of 3 and, in the same way, multiples of 9, will always be R9.

One way to prove that is given below[14]:

"Consider the natural number abcde. So $n=a\cdot10^4+b\cdot10^3+c\cdot10^2+d\cdot10^1+e\cdot10°$ with a≠0.

As 10^4 = 10000 = 9999+1104 = 10000 = 9999+1; 103 = 1000 = 999 + 1103 = 1000 = 999+1; 10^2 = 100 = 99+1; 10^1=10=9+1; we may rewrite n in the following form:
n=a·(9999+1)+b·(999+1)+c·(99+1)+d·(9+1)+e=9999·a+a+999·b+b+99·c+c+9·d+d+e, or yet:
n=3·(3333·a+333·b+33·c+3·d)+(a+b+c+d+e)
Now, if
k=3333·a+333·b+33·c+3·d, and t=a+b+c+d+e, then
n=3·k+t, with k,t ∈ N (i)
a) Suppose that n be divisible by 3.

Thus, n=3.x, for any natural number x and, in this way, by (i),
t=3·x−3·k=3·(x−k)t=3·x−3·k=3·(x−k).

Notice that n≥3·k, so z=x−k is a natural number. Thus, t=3·z, with z∈N and, then, t is divisible by 3.

But t=a+b+c+d+e, so, t is the sum of the algorithms of n; therefore this sum i divisible by 3.

It follows, if n is divisible by 3, then the sum of the algorithms of n is divisible by 3.
("n is divisible by 3"⇒⇒"sum of the algorithms of n is divisible by 3").

b)(⇐⇐) Suppose, now, that the sum of the algorithms of n is divisible by 3.

Then, t=a+b+c+d+e is divisible by 3 and, in this way, t=3·x, for any natural number x. Therefore, by (i), n=3·k+3·x=3·(k+x).

But k+x is a natural number, therefore, n is a multiple of 3, that is, n is divisible by 3.

By the example, if the sum of the algorithms of n is divisible by 3, then n is divisible by 3. ("the sum of the algorithms of n divisible by 3" ⇒⇒ "n divisible by 3").
Therefore, by **(a) and (b)**, it follows the criteria."

It can also be deduced, in the same way, that all the R3; R6 and R9 are multiples of 3.

As a corollary, it can be concluded too, that all the multiples of 9 are also R9.

A proof of that statement is given as follows[15]:

"The proof for the divisibility rule for 9 is essentially the same as the proof for the divisibility rule for 3.

For any integer x written as an· · · a3a2 a1a0 we will prove that if 9|(a0 + a1+ a2+ a3 ... + an), then 9|x and vice versa.
First, we can state that
x =a0+ a1×10 + a2×102+ a3×103... + an×10n
Next if we let s be the sum of its digits then
s = a0+ a1+ a2+ a3+ ... + an .
So
x - s = (a0- a0) + (a1×10 - a1) + (a2×102- a2) + ... + (an×10n- an)

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14. Translated from: http://clubes.obmep.org.br/blog/teoria-dos-numeros-um-pouco-sobre-divisibilidade-parte-2/um-pouco-sobre-divisibilidade-criterios-de-divisibilidade/ . Accessed on 06.29.19.
15. As seen at: https://sites.google.com/site/mathematicsnotebook/divisibilityrules/divisibility9.Accessed: 08/22/2019

= a1(10 - 1) + a2(102- 1) + ... + an(10n- 1).
If we let bk= 10k- 1, then bk = 9...9 (9 occurs k times) and bk = 9(1…1) and we can rewrite the previous equation as x - s = a1(b1)+ a2(b2)+ ... + an(bn)

It follows that all numbers bk are divisible by 9, so the numbers ak×bk are also divisible by 9. Therefore, the sum of all the numbers ak×bk (which is x-s) is also divisible by 9.
Since x-s is divisible by 9, if x is divisible by 9, then so is s and vice versa".

Immediately, it can be extracted from this demonstration, that all numbers of the reduction R3; R6; and R9 (except for 3) will never be prime numbers.

Equally, it can be realized that all numbers can be, then, represented as multiples of 9, in the following form: (9x+n), where n = nominator (1,2,3,4,5...9) and x= multiplicator.

In the other hand, there are some properties that the number families keep among themselves, which can be, equally, explored and used in other areas.

3 - PROPERTIES

3.1 – ADDITION

The first property of numeric reductions is the property of the sum of reductions, that implies that **the sum of any two numbers, shall have its reduction equal to the sum of the reduction of each one of these numbers.**

What has been said needs to be proven, and so it shall:

It has been said that all numbers can be expressed as multiples of 9, added to the nominator of the family of that number. In algebraic terms: **(9X+ nominator).**

It occurs that, as it was seen above, all numbers multiples of 9 are of the family R9, where they can be expressed as simply, 9X.

From this it can be extracted, then, that it will be the nominators that will characterize the family in which the number obtained by the sum shall fit.

So:

(9x+1) + (9x+2) = 18x+ 3 = 9(2x)+3, and the resulting number will belong to the family R3.

The same will happen with any other family that may be used, i.e., the numerals used in the adding to the multiples of 9 (here called nominators), will dictate the family to which the number, resulting from that sum, shall belong.

3.2 - MULTIPLICATION

It matters to note that, according to the fundamental theorem of arithmetic:

"*The fundamental theorem of arithmetic sustains that all positive integers greater than 1 can be decomposed as a product of prime numbers, being that decomposition unique least to the permutations of factors.".* (g.n.)[16]

Well, analyzing the theorem above stated, and the first premiss here introduced, it can be verified, then, that all non prime number (composite numbers), can be, therefore, represented in the following form:

(9a+x).(9b+y)= (9c+z)

where a; b and c are multiplier variables and x; y and z are nominators.

Similarly to the first statement, above proposed, the multiplicative property of reductions proposes that: **the result of the multiplication of any two numbers, shall have reduction equal to the multiplication of the reduction of each one of them.**

Now it shall be proven:

16.In: https://pt.wikipedia.org/wiki/Teorema_fundamental_da_aritm%C3%A9tica, access on 27/02/2107

It's been already said that all and any number can be described through the following formula: (9x+n), where n is the nominator.

According to the fundamental theorem of Arithmetic, every composite number can be described through the following formula: AB=C

Combining both, it results that:

A=(9a+x); B=(9b+y) and C= (9c+z), then

AB=C = (9a+x).(9b+y)= (9c+z)

Solving the multiplication of these factors the result is:

(81ab) + (9ay)+ (9bx) + xy = (9c+z)

Now, the first three elements of this function are all multiples of 9, whence they can be described, simply, as 9d, which implies that the equation will result as:

$$(9d+xy) = (9c+z)$$

From this, it can be extracted that:

As xy is the nominator of the element 9d+xy, and z is the nominator of the element 9c+z, and is the nominator that characterize the reduction itself, there follows that the second statement is proven.

4 – FACTORS OR FAMILY GENERATORS

Once these principles have been laid down, as above shown, there are enough tools to propose a simple and yet efficient solution to calculate prime numbers, in a precise manner.

In fact, beginning from the fundamental theorem of arithmetic (2.2 above), it follows that every number, not prime, can be expressed as a multiple of a prime number.

In the field of reductions, as proposed, it is also possible to verify that each family of numbers is generated by a limited group of factors belonging to pairs of pre-determined families.

Thus, a number, not prime, from family R2 (reduction base 2), can only be generated by a pair of factors from the families R1XR2; R4XR5 and R7XR8 and only in this conjugation (save, again, by the permutation of the position of the pairs).

Equally, the same can be said of all the other reduction families, that is, each family can only be the result of specific factors, belonging to families that can be previously determined.

The question then, is transported to knowing how to delimitate the families to which belong the factors capable of generating a number that belongs to each pre-determined family.

From a simple multiplication table it can be verified which numbers can be used as factors to produce numbers of each determined family.

Yes, for it must be remembered that the families keep a multiplicative behavior identical to the number that serves as nominator.

It’s only needed to look at a multiplication table to realize that the number families always keep the same pattern of possible factors.

In fact, one such a table would show that the composite numbers of the family R1 (those that can be expressed through the formula 9X+1) can only come from precise factors, which are, R1XR1; R2XR5; R4XR7 and R8XR8 (save, again, by the permutation of factor's positions). The same can be said of all other numerical families.

That is so, because, as the families follow exactly the same patterns of their nominators (as shown before), only those pairs of factors capable of generating these numbers (nominators), identical to the nominators that are sought, are the ones who might be the generators of each specific family.

Therefore, once we have visualized all the possible family interactions, it becomes clear which factors can generate each family.

From that same conclusion, it can be extracted that:

1) Non prime numbers of the family R1, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR1; R2XR5; R4XR7 and R8XR8**
2) Non prime numbers of the family R2, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR2; R4XR5 and R7XR8**

3) Non prime numbers of the family R3, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR3; R2XR6; R3XR4; R5XR6; R6XR8 and R7XR3;**
4) Non prime numbers of the family R4, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR4; R2XR2; R5XR8 and R7XR7;**
5) Non prime numbers of the family R5, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR5; R2XR7 and R4XR8.**
6) Numbers of the family R6, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR6; R2XR3; R3XR5; R4XR6; R6XR7 and R8XR3.**
7) Non prime numbers of the family R7, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR7; R2XR8; R4XR4 and R5XR5.**
8) Non prime numbers of the family R8, can only be obtained by the multiplication of pairs of factors in the following conjugation: **R1XR8; R2XR4 and R5XR7.**
9) Numbers of the family R9, can only be obtained by the multiplication of factors from any family by another factor of the R9 own family.

It follows logically, also, yet from that analysis, that the R3 family, whenever multiplied by any other factor, can only generate three numerical families, which are R3; R6 and R9, whence it can be extracted that, any number that belongs to one of these families is, necessarily, a multiple of 3.

From this, it can also be extracted that no number of these three families (except the number 3 itself), will be a prime number.

Finally, it can still be verified that the family R9, when used as a factor to produce any other number, will always produce a new number of the same family R9.

The remaining families, however, may contain (and in fact, do) prime numbers among their members.

Now it must be shown, then, how one can use this property to obtain prime numbers.

4.1 – DELIMITATING THE GENERATING FACTORS OF EACH FAMILY.

As any and all composite numbers can be expressed as the result of the multiplication of prime numbers (according to the fundamental theorem of arithmetic), conjugating this with the concepts proposed here, it becomes clear that any composite number can be expressed, then, as the multiplication of prime numbers that belong to the generating families, with potential to generate that specific number.

What we seek, then, is to determine which families of prime numbers can generate a specific number.

Realizing that the families of numbers can only be generated by specific families and in a predetermined order, shall make it easier to determine if a certain number is or not prime.

Beyond that, the use of such a tool, must allow the elaboration of simple formulas, capable of generating primes without failures or approximations.

5 – OBTAINING PRIME NUMBERS

As it has been seen above, the composite numbers of each families are obtained through the multiplication of predetermined factors.

Thus, by logical reasoning, all the numbers that belong to a certain family, but cannot be obtained by the multiplication of factors that belong to the generating families (as shown above), can only be prime numbers.

Written in algebraic terms, it becomes:

(9x+a) (9x+b) = (9x+c); but if (9x+a)(9x+b) ≠ (9x+c), this will be a prime number, as long as, “a” and “b” represent all the potential generators of the “c” family.

In this way, to obtain prime numbers of the R1 family, one would have the following formulas:

(9a+1)(9b+1) ≠ (9c+1)
(9a+2)(9b+5) ≠ (9c+1)
(9a+4)(9b+7) ≠(9c+1)
(9a+8)(9b+8) ≠ (9c+1)

primes of the family R1

Thus, to find out the first 10 prime numbers, belonging to the family R1, it should be enough to proceed to the following chart:

9c+1	(9a+1)(9b+1)	(9a+2)(9b+5)	(9a+4)(9b+7)	(9a+8)(9b+8)	PRIME/NOT
1	[9(0)+1][9(0+1)]	N/C	N/C	N/C	**N**
10	N/C	[9(0)+2][9(0)+5]	N/C	N/C	N
19	N/C	N/C	N/C	N/C	**P**
28	N/C	[9(0)+2][9(1)+5]	[9(0)+4][9(0)+7]	N/C	N
37	N/C	N/C	N/C	N/C	**P**
46	N/C	[9(0)+2][9(2)+5]	N/C	N/C	N
55	N/C	[9(1)+2][9(0)+5]	N/C	N/C	N
64	N/C	[9(0)+2][9(3)+5]	[9(0)+4][9(1)+7]	[9(0)+8][9(0)+8]	N
73	N/C	N/C	N/C	N/C	**P**
82	N/C	[9(0)+2][9(4)+5]	N/C	N/C	N
91	N/C	N/C	[9(1)+4][9(0)+7]	N/C	N
100	[9(1)+1][9(1)+1]	[9(0)+2][9(5)+5]	[9(0)+4][9(2)+7]	N/C	N
109	N/C	N/C	N/C	N/C	**P**
118	N/C	[9(0)+2][9(6)+5]	N/C	N/C	N
127	N/C	N/C	N/C	N/C	**P**
136	N/C	[9(0)+2][9(7)+5]	[9(0)+4][9(3)+7]	[9(0)+8][9(1)+8]	N
145	N/C	[9(3)+2][9(0)+5]	N/C	N/C	N
154	N/C	[9(0)+2][9(8)+5]	[9(2)+4][9(0)+7]	N/C	N
163	N/C	N/C	N/C	N/C	**P**
172	N/C	[9(0)+2][9(9)+5]	[9(0)+4][9(4)+7]	N/C	N
181	N/C	N/C	N/C	N/C	**P**
190	[9(1)+1][9(2)+1]	[9(0)+2][9(10)+5]	N/C	N/C	N
199	N/C	N/C	N/C	N/C	**P**
208	N/C	[9(0)+2][9(11)+5]	N/C	[9(0)+8][9(2)+8]	N
217	N/C	N/C	[9(3)+4][9(0)+7]	N/C	N
226	N/C	[9(0)+2][9(12)+5]	N/C	N/C	N
235	N/C	[9(5)+2][9(0)+5]	N/C	N/C	N
244	N/C	[9(0)+2][9(13)+5]	[9(0)+4][9(6)+7]	N/C	N
253	N/C	[9(1)+2][9(2)+5]	N/C	N/C	N
262	N/C	[9(0)+2][9(14)+5]	N/C	N/C	N
271	N/C	N/C	N/C	N/C	**P**
280	[9(1)+1][9(3)+1]	[9(0)+2][9(15)+5]	[9(0)+4][9(7)+7]	N/C	**N**

289	N/C	N/C	N/C	[9(1)+8][9(1)+8]	**N**
298	N/C	[9(0)+2][9(16)+5]	N/C	N/C	**N**
307	N/C	N/C	N/C	N/C	**P**

N/C = not contained

Table 1 – shows the first 10 primes of the R1 family

From the chart above, it is also possible to observe that the numbers follow patterns well defined, that is, all the four possible factor's equations, follow a simple arithmetic progression, what allows, then, the proposition of equations representatives of each one of these progressions, as follows:

a) the basic equation for the definition of number factors of each family, such as mentioned above, are the same (9a+x) (9b+y) = (9c+z)
b) the prime numbers, however, are defined, by the inequation: (9a+x).(9b+y) ≠ (9c+z)
c) as the only even number that is also a prime is 2, all other even factors, can be dismissed.

Therefore, one can say that, of the family R1, are primes all those odd numbers that, having reduction equal to 1, do not fit into the multiples of, at least, one of the following factors: R1XR1; R2XR5; R4XR7 and R8XR8.

Evidently, the use of the factors above will serve, primarily, as a test to verify if a determined number is or not a prime, going on an inverse way to the commonly proposed, specially when it relates to huge numbers.

At this point, the proposition presented, shows itself to be quicker and more efficient, since, besides reducing considerably the amount of operations (if compared to the process of simple divisors verification), it has the great advantage of having a precise answer, with no exceptions (as it happens with Fermat's small theorem and Miller-Rabin's tests).

It can be proposed then, that a given number n, to be verified if it is a prime or a composite number, one just has to follow these steps:

1) first, verify to which family it belongs – for that, it's enough to add all of its digits and repeat the operation until there is only one digit left.
2) once the family to which it belongs is found, then, use the factors inherent to this family, to verify if n is divisible by any one of these factors. If it is positive, then, the number in question is composite, if negative, it will be a prime number.

Obviously, this calculation of factors can be improved even further, limiting the checking of factors to only the ones that can produce the final digit of each specified number.

Thus, if a specific number ends with the digit 1, it's known that it can only be obtained by factors terminated in (1X1); (3X7) or (9X9). So, using these properties of the number families above proposed, one can search for the factors that, besides belonging to those groups that can generate the number in question, also keep symmetry to those other groups that can generate the number searched.

Below, these applications are checked as a test for the Lucas pseudo prime number, as a way of analyzing the advantages of the method here proposed:

Take the Lucas pseudo prime number 2737.

The sum of its algorithms indicate that it belongs to the family R1.

So, the possible factors of its production are: R1XR1; R2XR5; R4XR7 and R8XR8, as indicated in section 3, above.

As the number in question ends in 7, it's known that it can only be generated by factors that end in 1X7 or 3X9

So, it can be suggested the following method of verification:

R1XR1

[17]37X91= 3367
73X19 = 1387
73X109 = 7957

So, the genus R1XR1 of the family R1, can not generate such 163X19 = 3097 number. Let's see, now, the next genus (R2XR5).

R2XR5

11X47= 517
11X 137 = 1507
11X 227 = 2497
11X 317 = 3487
91X47 = 4277

So, neither this genus can generate the number searched. Let's pass, then, to the next genus.

R4XR7

31X7 = 217
121X7 = 847
211X7 = 1477
301X7 = 2107
391X7 = 2737

as the searched number is obtained, it is unnecessary to continue the operations.

Evidently, it will be easier to, simply, divide the number analyzed, than to plot so many multiplications.

This can be of considerable advantage, since many families of factors capable of generating the analyzed number, in many cases, are repeated, as in the case shown above in which there are the families R1XR1 and R8XR8.

In such circumstance, one could, simply, proceed in the following form:

Column R1XR1

2737/37 = 73,972972972...
2737/127= 21,5511811... [18]
2737/73 = 37,4931506849315068493150684931[19]

Column R2XR5

737/11 = 248,8181...
2737/91 = 30,07692...[20]
2737/47 = 58,23404...
2737/137 = 19,9781...[21]

Column R4XR7

2737/7 = 391 (factors reached)

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17. First prime number of the family R1 that ends in 7, times the first prime number of the family R1 that ends in 1 and is greater than 1.
18. the quotient is already smaller than the first number of the series of possible factors, so, this line can be abandoned, since it's proven that it will not be a factor of the searched number.
19. although still greater than the first number in the series (37), it is evident that the next quotient will already be smaller, and thus, this genus can also be abandoned.
20. for the same reason of the item above, it too can already be abandoned, since the first number of the other factor is 47.
21. it remains evident that this genus cannot generate the searched number.

As it can be seen, even pseudo prime numbers, as the one above, will be reached by the formulation here proposed, without any margin of error, being this its great advantage, not to mention the simplicity of the formulation.

6 – AMOUNT OF PRIME NUMBERS IN A GIVEN INTERVAL

To count the amount of prime numbers in a given interval, using the propositions above sustained, one can use simple arithmetic progression equations, as it will be shown below.

As it had been shown above, all numbers can be written as multiples of 9, or (9x+n).

Logically, if one wants to find out how many prime numbers of a certain family there are in a given interval, it would be possible to do it, as follows:

Let there be a number interval between numbers a and b; in which, one desires to find out how many prime numbers of the family R2 exist.

Since R2 = (9x+2), and that R2 = R1XR2 + R4XR5 +R7XR8 + All R2 primes (PR2), it follows that:

Σ PR2 = Σ R2 - (Σ R1XR2 + Σ R4XR5 + Σ R7XR8)

It must also be noted that some numbers will be produced by more than just one pair of factors, which could be called duplicate (or DR2), and these, too, must be excluded from the count.

Thus,

Σ PR2 = Σ R2 - (Σ R1XR2 + Σ R4XR5 + Σ R7XR8 – DR2)

For example, if one tries to find out how many prime numbers, belonging to the R2 family there are in the interval from 1 to 1001, it could be done as follows:

First, since all primes (except 2) are odd numbers, it can be stipulated that instead of expressing numbers as (9X+n), it could be expressed as (18X+n), as a way of expressing only the odd numbers, as long as the first n is also the first odd number belonging to the family in question.

So, returning to the task at hand, since the smallest R2 odd number is 11, we'd have:
(18X+ 11)= R2.

The search should then, begin with the factors R1XR2, which implies that, since 1001/11= 91, it means that this is biggest R1 number that shall be used to find the factors sought.

In the other hand, since 1001/19 (the first R1 odd number in the interval) = 52,68..., it means that the greatest R2 number that shall be used as a factor is 47 (the next odd R2 number would be 65, which is greater than 52)

It can also be noted that it forms an arithmetic progression whose ratio is 18x.

So, if we take the general equation for arithmetic progression:

an = a1+ (n - 1).r , it leaves us, with the following:
first term = 11X19 = 209, then
n-1 = (1001 – 209)/ (11X18)
n = 5

Therefore, there will be 5 terms for this progression, or 5 numbers generated by the factors 11XR1 in the interval 1 to 1001.

Applying the same equation to the other factors of the this genus (R1XR2), one shall have:
29 X R1 = 1 factor [1001 – (29x19)] /(29x18) = 0,862..., thus it only has the first term
47 X R1 = 1 factor

Since 47 was already established as the last one that would be possible, we can securely say that **there are 7 terms produced by the genus R1XR2** in this interval.

There are also two more genus to consider: R4XR5 and R7XR8.

Applying the same rule to these, we'd come up with
1001/5 = 200,2, which implies that 193 is the greatest R4 odd number to be used.

Therefore
1001 – 65 = 936
936/ (18X13) = 936/234 = 4, thus, there are 5 terms for the species 13R5

As for the remaining species:

31R5 = 2 terms
49R5 = 1 term
67R5 = 1 term
85R5 = 1 term
103R5 = 1 term
121R5 = 1 term
139R5 = 1 term
157R5 = 1 term
175R5 = 1 term
193R5 = 1 term

So there are 16 terms of this genus (R4XR5).

Finally, the genus R7XR8:

7R8 = 8 terms
25R8 = 2 terms
43R8 = 1 term

Thus, this genus has 11 terms.

But it still remains the need to eliminate the duplicity, for some numbers may be generated by more than one genus.

The duplicity occurs whenever a number of the generating families is itself a composite number generated by one of the other generating families.

In this case, since it is the R2 family that is being searched, one can say that all of its numbers have been produced already, so there's need to focus on the other generating families, namely, R1; R4;R5;R7 and R8.

Any duplicity needs to come from the interaction between these other families.

Beginning with family R1, it has been shown that it can be generated by these factors:

R1XR1; R4XR7; R8XR8 and R2XR5.

Since the greatest possible R1 factor used to produce numbers in this interval is 91, one needs to look only for the R1 numbers smaller than 91. In other words, there's need to search only for R1≤91.

Since 1 always results in the same number whenever used as a factor, it can be dismissed, which implies that 19 will be considered as the smallest R1 odd number factor. Now, since 19 X19 = 361 and 91<361, therefore, there are no R1XR1 factors that could generate R2 family numbers in this interval.

As for R4XR7 genus:

The smallest R4 odd number in this interval is 13 and the smallest R7 is 7.

Now, 7X13= 91, which is the greatest R1 number used as a factor, thus, this genus (R7XR4) has only one duplicity in this interval.

Similarly, the genus R8XR8 has, as its smallest number, 17, it turns out that, since 17X17 = 289, and since 289> 91 (which is the greatest R1 number used to generate numbers in this interval), there are no R8XR8 duplicate factors in this interval.

Finally, the R2XR5 genus:

Since 11 is the first R2 odd number, and 5 is the smallest R5:

55 is the first R1 produced by the R2XR5 genus.

Now, 55X11 = 605, thus, this is the first duplicity.

Because (29X5) = 145 and (11X23) = 253, which are both > 91, it can be stated that there's only one duplicity in this interval, due to the genus R2XR5.

Therefore, the family R1 has 2 duplicate R2 generated numbers, namely, 605 (55 X 11) and 1001 (91X11).

Applying the same method to the other families, we'd arrive at the chart bellow:

Family	Composite factors	Duplicate R2
R4	49; 85; 121; 175	245; 425; 605; 875
R5	77	1001
R7	25	425;875
R8	35; 125; 143	245; 875; 1001
R1	55; 91	605; 1001

Total 12-5 = 7

Table 2 – shows the composite factor of each family and the duplicate numbers that exists up to 1001.

Obviously, the duplicate numbers must be counted only once, what means that out of the 12 duplicate, only 5 shall be accounted for, and the other 7 shall be subtracted from the rest.

The amount of R2 numbers, can be discovered through the arithmetic progression equation, as follows:

an = a1+ (n - 1).r
1001 = 11 + (n-1) 18
990 = (n-1) 18
55 = n-1
56 = n

As the number 2 is also of the family R2, it should be added to these, as well, so that the total amount is 57.

Knowing all these data, the original equation can, now, be solved.

Σ PR2 = Σ R2 - (Σ R1XR2 + Σ R4XR5 + Σ R7XR8 - DR2)
Σ PR2 = 57 – (7 + 16 + 11 - 7)
Σ PR2 = 57 - 27
Σ PR2 = 30

So, there are 30 prime numbers of the family R2 in the interval from 1 to 1001.

As it is logical, if there was a need to find out the amount of all the prime numbers contained in the same interval, it would be necessary to do the same proceedings as above, with all of the other families of numbers, but the equation would remain the same, changing only the elements pertaining to each family.

7 – FINDING PRIMES THAT END IN A CERTAIN DIGIT.

Differently from any other method ever tried, using the properties here discussed, allows the search for prime numbers that end in a certain digit, and more than that, that belong to a certain family and end on that precise digit.

Taking the example above, suppose one was to find what were the first 10 primes, which belong to the R2 family and end on 1 (or the first 10 primes of the species $R_1$1).

First it is needed to take the first R2 odd number that ends on 1, which is 11.

As it has been shown above, at every 90 numbers there will be another number of the same family ending on the same digit.

So, it could be stated as (11+90X).

Now, the smallest R1XR2 number that ends on 1, is 551 (or 19X29), so every R2 < 551, that ends on 1, will not be generated by the genus R1XR2.

In the same manner, the smallest R4XR5 number that ends on 1 is 1001, so every R2<1001, that ends on 1, will not be generated by the genus R4XR5.

And in the gender R7XR8, the smallest number that ends on 1 is 371, so every R2<371, that ends on 1, will not be generated by the genus R4XR5.

Finally, it must be remembered, as was shown above, that only factors terminated on 1X1; 2X5 or 3X7 (save for the alteration of position) will be able to generate numbers ending on 1.

That would leave the following chart:

R2 (11+90x)	R1XR2	R4XR5	R7XR8	P/N
11				P
101				P
191				P
281				P
371			371 (7X53)	N
461				P
551	551 (19X29)			N
641				P
731			731 (17X43)	N
821				P
911				P
1001	1001 (11X91)	1001(13X77)	1001 (7X143)	N
1091				P
1181				P

Table 1 – shows the first 10 primes of the R2 family and species 1 (who ends on 1).

So, the numbers 11; 101; 191; 281; 461; 641; 731; 821; 911; 1091 and 1181, are the first ten primes of the family R2, that end on 1, and 1181 is the 10th R2 prime that ends on 1.

CONCLUSION

Through the method above explained, it is possible to obtain prime numbers of any kind or size, with certainty, and no need to check them individually.

Equally, these same properties may be used to verify if a determined number is or not a prime.

The proposition, now at hand, has as its advantage, the use of mathematical methods sufficiently simple, as to make the search for prime numbers quick and efficient.

The analysis of the digits that compose a number also follow a defined pattern, whose properties may be used for various purposes, including (as it is hoped to have been demonstrated above), to obtain prime numbers, as well as to establish the families to which they belong.

These properties inform the behavior of numbers according to their families, genus and species, in each one of the basic operations, showing how different numbers shall have the same results, when subjected to mathematical operations, as long as they are analyzed under the lenses of their numerical properties.

This number similarity can be, then, scrutinized according to categories, that are named families (the reduction of the number), genus (which family of factors can generate the number) and species (the digit on which the number ends), according to the similarity of each sub-group.

The analysis of these properties may be used with success for the resolution of problems, as the obtaining or verifying of prime numbers of any orders, simply and quickly enough, to make it viable, using only common elements of basic arithmetic.

Also, they allow the operation of finding primes that end on certain digits, which could not be done with any of the other previous methods.

Utterly, as all the operations used in the method here proposed are simple enough as to be taught to elementary school students, it is believed that the use of such way of searching, would be much easier than the usual methods today available, serving as a practical tool and dismissing the use of more complex and difficult methods such as Riemann's Zeta function.

Given the simplicity and easiness of this method, it is believed that the search for primes has now a better and more efficient tool, ready to be used and taught to others.

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