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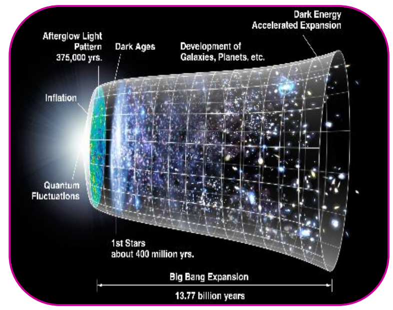
STRING COSMOLOGICAL PERFECT FLUID MODEL WITH ELECTROMAGNETIC FIELD AND COSMOLOGICAL CONSTANT

Dr. Milan M. Sancheti
Department of Mathematics, R. A. Arts, M. K. Commerce and
S. R. Rathi Science College Dist. Washim (M.S), India.
Corresponding Author: msancheti7@gmail.com

ABSTRACT

We have investigated the String Cosmological perfect fluid model with cosmological constant, the perfect fluid is taken to be one obeying us all of state (EOS) P = γρ. I used $T = {}^{E}T_{ij} + {}^{S}T_{ij}$ *of stress energy tensor where* ${}^{E}T_{ij}$ *is the energy momentum tensor for electromagnetic field and* ${}^{S}T_{ij}$ *is the energy momentum tensor for string cloud. We have also studied the string cosmological model by considering the time dependent deceleration parameter with the presence of electromagnetic field. Some cosmological parameters are also discussed and studied.*



KEY WORD: *String Cosmological Model, Perfect Fluid, Cosmological Constant, Electromagnetic Field.*

INTRODUCTION

The recent scenario of early inflation and late time accelerated expansions of the universe (Rises et al. 1998; Perlmutter et al. 1999) is not explained by general theory of relativity. Hence to incorporate the above desirable features there have been several modifications of general relativity. Significant among them are scalar-tensor theories of gravitation formulated by Brans and Dicke (1961). In 1961, Brans and Dicke formulated the scalar-tensor theories of gravitation on the basis of coupling between an adequate tensor field and scalar field ϕ. The scalar field has a dimension of G – 1where G is the gravitational constant. Therefore, ϕ−1play the role of G(t). This theory successfully describes the Mach's principle but fails toexplain the missingmatter problems and absolute properties of space. Later on Saezand Ballester (1985) developed a scalar-tensortheory in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In recent years there has been an immense interest in constructing cosmological models of the universe to study the origin, physics and ultimate fate of the universe. In particular, cosmological models of Brans-Dicke and Saez- Ballester scalar-tensor theories of gravitation are attraction more and more attention of scientists. Brans-Dicke (1961) theory is a well known competitor of Einstein's theory of gravitation. It is the simplest example of a scalar-tensor theory in which the gravitational interaction is mediated by a scalar field ϕ as well as the tensor field g_{ij} of the Einstein's theory. In this theory, the scalar field ϕ has the dimension of the universal gravitational constant. So in literature, manyauthors consider plane symmetry, which is less restrictive than spherical symmetry and provides an avenue to study homogeneities in early universe. Da Silva and Wang (1998), Anguige (2000), Nouri-Zonoz and Tavanfar (2001), Pradhan et al. (2003, 2007) and Yadav (2011) have studied the plane symmetricand homogeneous cosmological models in different physical context.

In this paper, we find String Cosmological perfect fluid model with cosmological constant. The paper is organized as follows:

1. We have provided the metric and field equation inconnection to the proposed model for homogeneous plane symmetric model Electromagnetic Field
2. We used $T = {}^{E}T_{ij} + {}^{S}T_{ij}$ of stress energy tensor where ${}^{E}T_{ij}$ is the energy momentum tensor for electromagnetic field and ${}^{S}T_{ij}$ is the energy momentum tensor for string cloud.
3. We have provided the physical properties of the model.

Basic Equations

We consider the plane symmetric metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \quad (1)$$

Where, the metric potential A & B are the function of x & t alone.

The energy momentum tensor of the mass less scalar field:

The energy momentum tensor for the matter under the discussion is given by

$$T_{ij} = {}^{S}T_{ij} + {}^{E}T_{ij} \quad (2)$$

Where ${}^{S}T_{ij}$ is the energy momentum tensor for string cloud and ${}^{E}T_{ij}$ is the energy momentum tensor for electric magnetic field. The energy momentum tensor for string cloud for Perfect Fluid is given by

$$T_{ij} = (\rho - P)\, u_i u_j - \lambda\, x_i x_j \quad (3)$$

With $u_i u^j = - x_i x^i = -1$, $x^i u_i = 0$

Where ρ is the rest energy density of the cloud of strings with particles attached to them and λ is the tension density of the string, u_i is the cloud four velocities and x_i is the direction of anisotropy. In the commoving coordinates the magnetic field is taken along ϕ direction. So that only non vanishing components of electromagnetic field tensor F_{ij} is F_{12}.

Therefore F_{12} = constant = H (say) = F_{21}

$$F^{12} = g^{1\alpha} g^{2\beta} F_{\alpha\beta} . \quad (4)$$

We consider $\rho = \rho_p + \lambda$

Where ρ_p is the density of the cloud of string,

The electromagnetic energy momentum tensor is ${}^{E}T_{ij} = 1/4\pi\, [- F_{is} F_{jp} g^{sp} + ½\, g_{ij} F_{sp} F^{sp}]$ (5)

where F_{ij} is the electromagnetic field tensor derived from potential ϕ_j defined as

$$F_{ij} = \phi_{i,j} - \phi_{j,i} \quad (6)$$

So the first set of Maxwell equation

$$F_{ij,k} + F_{jk,I} + F_{ki,j} = 0 \quad (7)$$

Here F_{12} = constant = H and other all components are zero. from equation 3 and 5 the non vanishing components of ${}^{S}T_{ij}$ and ${}^{E}T_{ij}$ can be respectively written as

${}^{S}T_1^{\ 1} = {}^{S}T_2^{\ 2} = 0,\ {}^{S}T_3^{\ 3} = -\lambda,\ {}^{S}T_4^{\ 4} = -\rho + P$ (8)

and ${}^{E}T_1^{\ 1} = {}^{E}T_2^{\ 2} = -\,{}^{E}T_3^{\ 3} = -\,{}^{E}T_4^{\ 4} = H^2/\,8\pi A^2B^2$ (9)

The Einstein field equation are $G_i^j = R_i^j - 1/2\ \delta_i^j\ R$

The homogeneous field equation

$$-8\pi T_i^j = R_i^j - \frac{1}{2}R\delta_i^j - \Lambda\delta_i^j \tag{10}$$

For line element (1) are

$$\frac{\dot{B}^2}{B^2} + \frac{2\ddot{B}}{B} = \frac{H^2}{A^2B^2} + \Lambda \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = \frac{H^2}{A^2B^2} + \Lambda \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{B}}{B} = 8\pi\lambda - \frac{H^2}{A^2B^2} + \Lambda \tag{13}$$

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB} = 8\pi(\rho - P) - \frac{H^2}{A^2B^2} + \Lambda \tag{14}$$

Subtracting (11) from (12), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0 \tag{15}$$

Let V be a function of ‘t’ denoted by

$$V = A^2B \tag{16}$$

Then from (15), we obtain

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} = 0 \tag{17}$$

By integrating above equation, we get

$$\frac{A}{B} = L_2 \exp\left(L_1 \int \frac{1}{V}dt\right) \tag{18}$$

Where L_1 & L_2 are the constants of integration.

In view of

$$V = AB^2 \quad \rightarrow \quad A = \frac{V}{B^2}$$

We can write A, B in the explicit form

$$A = D_2V^{\frac{1}{3}}\exp\left(X_2 \int \frac{1}{V}dt\right) \tag{19}$$

$$B = D_1V^{\frac{1}{3}}\exp\left(X_1 \int \frac{1}{V}dt\right) \tag{20}$$

Where D_i (i = 1,2) &X_i (i = 1,2) satisfy the relation

$$D_1^2D_2 = 1 \quad \text{and} \quad 2X_1 + X_2 = 0$$

Adding the equations (11), 2 times of (12) and 3 times of (14) we yield

$$\frac{2\dot{B}^2}{B^2} + \frac{2\ddot{B}}{B} + \frac{4\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = 12\pi(\rho - P) + 3\Lambda \tag{21}$$

In view of $V = A\,B^2$, from (21) we obtain

$$\frac{\ddot{V}}{V} = 12\pi(\rho - P) + 3\Lambda \tag{22}$$

The conservation law for the energy momentum tensor gives

$$\dot{\rho} = -\frac{\dot{V}}{V}(\rho + P) \tag{23}$$

From (22) and (23), we have

$$\dot{V} = \sqrt{24\pi V^2 \rho + 3\Lambda V^2 + 2C_1} \tag{24}$$

Where C_1 being integration constant.
Rewriting (22) in the form

$$\frac{\dot{\rho}}{\rho + P} = -\frac{\dot{V}}{V} \tag{25}$$

And taking into account that the pressure and the energy density obeying an equation of state of type $P = f(\rho)$, we conclude that ρ & P are the function of V.
Hence the right hand side of (21) is a function of V only

$$\ddot{V} = 12\pi(P - \rho)V + 3\Lambda V = F(V) \tag{26}$$

From the mechanical point of view equation (26) curve interpreted as equation of motion of single particle with unit mass under the force F(V), then

$$\dot{V} = \sqrt{2(\epsilon - U(V))} \tag{27}$$

Here ϵ can be viewed as energy and $U(V)$ as the potential of the force F.
Comparing (24) and (27), we find

$$\epsilon = C_1 \ \& \ U(V) = -[12\pi V^2 \rho + \frac{3}{2}\Lambda V^2] \tag{28}$$

Finally, we write the solution of (24) in the quadrature form

$$\int \frac{dV}{\sqrt{2(C_1 + 12\pi V^2 \rho + \frac{3}{2}\Lambda V^2)}} = t + t_0 \tag{29}$$

Where, the integration constant t_0 can be taken to be zero, since it only gives a shift in time. Hence, let us take $t_0 = 0$.

$$\int \frac{dV}{\sqrt{2(C_1 + 12\pi V^2 \rho + \frac{3}{2}\Lambda V^2)}} = t \tag{30}$$

Universe as a binary mixture of perfect fluid

We consider the evolution of the plane of symmetric universe filled with the perfect fluid

$$P = \gamma\rho \tag{31}$$

Here the γ is constant and lies in the interval$\gamma \ \epsilon \ [0,1]$. Depending on its numerical values γ describes the following types of universe

$$\left.\begin{array}{l}\gamma = 0 \text{(Dust universe)} \\ \gamma = 1 \text{(Zeldovich Universe or Stiff matter)}\end{array}\right\} \quad (32)$$

In a co-moving frame the conservation law of energy momentum tensor leads to the balance equation for the energy density

$$\dot{\rho} = -\frac{\dot{V}}{V}[\,\rho + P] \quad (33)$$

From the equation (3.3) we get,

$$P = \frac{\gamma\rho_0}{V^{1+\gamma}} \quad (34)$$

&

$$\rho = \frac{\rho_0}{\gamma\, V^{1+\gamma}} \quad (35)$$

Here ρ_0 is the integration constant.
Therefore, equation (30) gives

$$\int \frac{dV}{\sqrt{2C_1 + \frac{24\pi\rho_0 V^{1-\gamma}}{\gamma} + 3\Lambda V^2)}} = t \quad (36)$$

Case I:

Let us consider $\gamma = 1$ (Zeldovich universe) for $C_1 = 0$, equation (3.6) reduces to

$$\int \frac{dV}{\sqrt{\left(\sqrt{3\Lambda V}\right)^2 + \left(\sqrt{24\pi\rho_0}\right)^2}} = t \quad (37)$$

This gives,

Spatial Volume V:

$$V = \frac{\sqrt{8\pi\rho_0}}{\sqrt{\Lambda}} \sinh\sqrt{3\Lambda}\, t \quad (38)$$

Putting the value of V in eq^n (19) and (20), we get

$$\left.\begin{array}{l} A = D_2 (K_1)^{\frac{1}{6}} \sinh^{\frac{1}{3}}\left(\sqrt{3\Lambda t}\right)\left(\mathrm{cosech}\left(\sqrt{3\Lambda}\, t\right) + \coth\left(\sqrt{3\Lambda}\, t\right)\right)^{\frac{X_2}{\sqrt{3\Lambda}K_1}} \\ B = D_1 (K_1)^{\frac{1}{6}} \sinh^{\frac{1}{3}}\left(\sqrt{3\Lambda t}\right)\left(\mathrm{cosech}\left(\sqrt{3\Lambda}\, t\right) + \coth\left(\sqrt{3\Lambda}\, t\right)\right)^{\frac{X_1}{\sqrt{3\Lambda}K_1}} \end{array}\right\} \quad (39)$$

Where$K_1 = \sqrt{\frac{8\pi\rho_0}{\Lambda}}$ &$D_i(\, i = 1,2)$&X_i $(i = 1,2)$ satisfy the relation

$$D_1^2 D_2 = 1 \qquad \& \qquad 2X_1 + X_2 = 0$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter Δ, the shear scalar B^2 and the deceleration parameter q.
They are defined as

$$\theta = 3H \quad (40)$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \quad (41)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) = \frac{3}{2}\Delta H^2 \quad (42)$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \quad (43)$$

By using above equations, we can express the physical quantities as
From equation (34) and (38), we have

Rest energy density ρ:

$$\rho = \frac{\Lambda}{8\pi \sinh^2(\sqrt{3\Lambda}\, t)} \tag{44}$$

i.e.,

Pressure P:

$$P = \rho = \frac{\Lambda}{8\pi \sinh^2(\sqrt{3\Lambda}\, t)} \tag{45}$$

Expansion Tensor:

$$\theta = \sqrt{3\Lambda} \coth \sqrt{3\Lambda}\, t \tag{46}$$

Anisotropy Parameter:

$$\Delta = \frac{2}{9H^2}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2 \tag{47}$$

Shear Scalar:

$$\sigma^2 = \frac{1}{3}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right]^2 \tag{48}$$

Where A and B by using equation (39)

The deceleration parameter q:

$$q = 3\,\mathrm{sech}^2(\sqrt{3\Lambda}\, t) - 1 \tag{49}$$

The tensor density λ of the string can be obtained as:

Subtracting eq^n 12 from 13 we get

$$\lambda = \frac{H^2}{4\pi A^2 B^2} \tag{50}$$

Case II:

Let us consider $\gamma = 0$ (Dust universe) for $C_1 = 0$, equation (36) reduces to t = 0. It is not possible.

CONCLUSION

In this paper, we have studied the String cosmological perfect fluid model with Electromegnetic field and cosmological Constant as source of matter within the framework of scalar-tensor theory of gravitation. The exact solution to the corresponding filed equation is found. The inclusion of the perfect fluid into the system gives rise to an accelerated expansion of the model. The particular cases for $\gamma = 1$ have also been analysed in detail.

It is observed that the following result

- For this model, we observed that the spatial value is constant at t = 0.
- Scalar expansion and shear scalar and average scalar factor tends to infinity as t = 0.
- Declaration parameter q is constant at t = 0.
- The tensor density is constant.
- The Pressure tends to infinite at t = 0
- Rest energy density ρ tends to infinite at t = 0
- Hence in absence of electromagnetic field along z axis the geometric or other string does not exist.

The particular cases for $\gamma = 0$, It is not Possible to find result.

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