REVIEW OF RESEARCH
IMPACT FACTOR : 5.7631(UIF) UGC APPROVED JOURNAL NO. 48514 ISSN: 2249-894X

VOLUME - 8 | ISSUE - 1 | OCTOBER - 2018

TOLERANCE INTERVAL FOR A K-UNIT PARALLEL SYSTEM BASED ON TYPE-II CENSORED DATA FROM GUMBEL DISTRIBUTION

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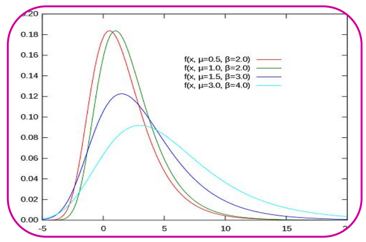
ABSTRACT

In this paper, we consider a k-unit parallel system with component lifetime distribution is Gumbel. Based on progressively Type-II censored sample, the maximum likelihood estimator (MLE) of the parameter of Gumbel distribution is derived. An approximate β-expectation tolerance interval (TI) is constructed. In order to compare the performance of the tolerance interval, we conduct simulation experiment.



KEYWORDS: *Progressively Type-II censoring, MLE, β-expectation TI.*

1. INTRODUCTION-

In life testing experiments, it is impossible to observe lifetimes of all the units because of time limits and other restrictions. In such a situation, censoring is a common practice. Broadly censoring is classified into two types; Type-I and Type-II censoring.

Type-II censoring depends on number of failures. That is, experiment continues up to the pre-specified number of failures. For example, in life testing experiment, suppose n units are placed on test and the test is terminated at the failure time of the m^{th} ($m \leq n$) unit instead of continuing until all n units to fail. In case of Type-II censoring, the number of observations with exact lifetimes is fixed.

In conventional Type-II censoring, all the remaining survived units are removed from the experiment at termination point. In progressive censoring scheme, some survived units may be removed at different stages rather than the final termination point only. Progressive censoring scheme is applied in both Type-I and Type-II censoring scheme.

In progressive type-II censoring scheme, suppose n units are put on test and we observe only m units. The number m and $R_1, R_2, \ldots, R_m$ are specified prior to the test. Since $\sum_{i=1}^{m} R_i = n - m$. At the time of first failure R_1 units are removed from remaining n-1 units. At the time of second failure R_2 units are removed from remaining $n-2-R_1$ units and so on. At the m^{th} failure all remaining R_m units are removed from the experiment. Here, we observe failure time of m units and remaining n-m units are removed from the experiment at different stages of experiment. In conventional Type-II censoring scheme $R_1=R_2=\ldots= R_{m-1}=0$ and $R_m= n-m$. In this paper, we consider the progressive Type-II censoring scheme for estimation purpose.

Progressive Type-II censoring scheme for various lifetime distributions is discussed in various articles. Cohen (1963) studied MLE of the parameters of exponential and normal distribution for progressively Type-II censored samples. Mann (1969, 1971) considered estimation on Weibull distribution with progressive censored samples. Balakrishnan et. al. (2003, 2004) discussed inference for Gaussian and extreme value distribution under progressive Type-II censoring scheme. Ng (2005) studied parameter estimation for modified Weibull distribution for progressive Type-II censored samples. Balakrishnan and Aggarwala (2000) gave details about progressive Type-II censoring scheme.

Pradhan (2007) considered point and interval estimation of a k-unit parallel system based on progressive Type-II censoring scheme with exponential distribution as the component lifetime distribution. Chein and Balakrishnan (2011) studied consistency and asymptotic normality of maximum likelihood estimator (MLE) based on progressive Type-II censored samples. Potdar and Shirke (2014) discussed inference for the scale family of lifetime distributions based on progressively censored data.

In this article, we discuss estimation of the parameter of a k-unit parallel system based on progressive Type-II censoring scheme with Gumbel distribution as the component life distribution. Estimation of parameter is based on the lifetimes of the system. We study tolerance interval (TI) for the lifetime of system, on the lines of Kumbhar and Shirke (2004).

MLE has important properties like consistency, uniqueness, invariance etc. MLE is also function of sufficient statistic. Therefore, we used maximum likelihood estimation method to estimate parameter and function involved in the model. This estimation procedure is discussed in Section 2. β-expectation TI derived in Section 3. To investigate performance of procedure, simulation study is carried out in Section 4. Also, we compare simulated and approximate expected coverage. Finally, conclusions are reported in Section 5.

2. ESTIMATION –

Consider Gumbel distribution with location parameter μ= 0 and scale parameter 1/λ. The probability density function (pdf) $g(x;\lambda)$ and cumulative density function (cdf) $G(x;\lambda)$ are respectively given by,

$$g(x;\lambda) = \lambda e^{-\lambda x}\, exp\left(-e^{-\lambda x}\right) \qquad -\infty < x < \infty, \lambda > 0. \qquad (1)$$
$$G(x;\lambda) = \exp\left(-e^{-\lambda x}\right) \qquad -\infty < x < \infty, \lambda > 0. \qquad (2)$$

Consider k-unit parallel system with independent and identically distributed components. Let X_1, X_2,....., X_k be the lifetimes, where X_i is the lifetime of the i^{th} component having pdf $g(x_i;\lambda)$. Lifetime of the system X= max(X_1, X_2, ... , X_k). The cdf and pdf of X is

$$F(x;\lambda) = [\exp\left(-e^{-\lambda x}\right)]^k \qquad -\infty < x < \infty,\ \lambda > 0, \qquad (3)$$
$$f(x;\lambda) = k\lambda\, [\exp\left(-e^{-\lambda x}\right)]^k\, e^{-\lambda x} \qquad -\infty < x < \infty,\ \lambda > 0. \qquad (4)$$

Suppose n observations are taken on the system. Let $x_{(1)}$, $x_{(2)}$, ... , $x_{(n)}$ be the order statistics from a progressively type-II censored sample of size n with (R_1, R_2,..., R_m) progressive censoring scheme. The likelihood function is

$$L(x|\lambda) = C\prod_{i=1}^{m} f(x_{(i)};\lambda)\,[1 - F(x_{(i)};\lambda)]^{R_i}, \qquad \text{where } C = n\prod_{j=1}^{m-1}\left(n - j - \sum_{i=1}^{j} R_i\right)$$
$$L(x|\lambda) = C\prod_{i=1}^{m} k\lambda\,[\exp\left(-e^{-\lambda x_{(i)}}\right)]^k\, e^{-\lambda x_{(i)}}\, \left\{1 - [exp\left(-e^{-\lambda x_{(i)}}\right)]^k\right\}^{R_i} \qquad (5)$$

Differentiating log-likelihood function with respect to λ and equate it to zero, we get

$$\frac{dlogL}{d\lambda} = \frac{m}{\lambda} - \sum_{i=1}^{m} x_{(i)} + \sum_{i=1}^{m} x_{(i)} y_{(i)} - k\sum_{i=1}^{m} R_i x_{(i)} \frac{[\exp(-y_{(i)})]^k y_{(i)}}{1 - [\exp(-y_{(i)})]^k} = 0, \qquad (6)$$

where $y_{(i)} = e^{-\lambda x_{(i)}}$

To compute MLE of λ, we have to solve equation (6) for λ. But, solution of this equation for λ is not in closed form; therefore, we use Newton-Raphson method for obtaining MLE. Least square estimate is used as an initial value of λ into the Newton-Raphson method. Ng (2005) discussed estimation of model parameters of modified Weibull distribution based on progressively Type-II censored data where the empirical distribution function is computed as (see Meeker and Escober 1998)

$$\hat{F}\left(x_{(i)}\right) = 1 - \prod_{j=1}^{i}(1 - \hat{p}_j),$$

$$with\ \hat{p}_j = \frac{1}{n - \sum_{k=2}^{j} R_{k-1} - j + 1}\ for\ j = 1,2,\dots,m$$

The estimate of the parameters can be obtained by least squares fit of simple linear regression

$y_i=\beta x_{(i)}$ with $\beta= -\lambda$

$$y_i = \log\left\{-\frac{1}{k}\log\left[\frac{\hat{F}(x_{(i-1)})+\hat{F}(x_{(i)})}{2}\right]\right\} \qquad \text{for } i=1,2,\dots,m$$

$$\hat{F}\left(x_{(0)}\right) = 0$$

The least square estimate of λ is given by

$$\hat{\lambda} = -\frac{\sum_{i=1}^{m} x_{(i)} y_i}{\sum_{i=1}^{m} x_{(i)}^2} \tag{7}$$

For simulation study, we use $\hat{\lambda}$ as an initial value into the Newton-Raphson method to obtain MLE.

3. TOLERANCE INTERVALS –

Kumbhar and Shirke (2004) derived the expression for β-expectation TI for the lifetime distribution of a k-unit parallel system with component life as exponential distribution. They investigate the performance of the TI based on complete data. Pradhan (2007) studied the performance of the TI based on progressively Type-II censored data from exponential distribution. We study the performance of the TI based on progressively Type-II censored data from Gumbel distribution. Let $l_\beta(\lambda)$ be the lower quantile of order β of the distribution function F(x; λ). Then, we have

$$l_\beta(\lambda) = -\frac{1}{\lambda}\log\left[-\frac{1}{k}\log(\beta)\right]$$

Thus, an upper β-expectation tolerance interval for F(x; λ) is obtained by

$$I_\beta = \left(0,\ l_\beta(\lambda)\right)$$

The maximum likelihood estimate of $l_\beta(\lambda)$ is given by

$$l_\beta(\hat{\lambda}_n) = -\frac{1}{\hat{\lambda}_n}\log\left[-\frac{1}{k}\log(\beta)\right]$$

yielding the approximate β- expectation tolerance interval

$$\hat{I}_\beta = \left(0,\ l_\beta(\hat{\lambda}_n)\right)$$

The expectation of $\hat{I}_\beta$ can be obtained approximately by using the approach suggested below.

$$E\left[F\left(l_\beta\left(\hat{\lambda}_n\right);\lambda\right] \approx \beta - o.5F_{02}\sigma^2\left(\hat{\lambda}_n\right) + \frac{F_{01}\sigma^2\left(\hat{\lambda}_n\right)F_{11}}{F_{10}}, \quad (8)$$

$$where\ F_{10} = \frac{\partial F}{\partial x,}, F_{01} = \frac{\partial F}{\partial \lambda,}, F_{11} = \frac{\partial^2 F}{\partial x \partial \lambda,}, F_{02} = \frac{\partial^2 F}{\partial \lambda^2,}$$

$$F_{10} = \lambda kg[\exp(-g)]^k, \quad F_{01} = \lambda xg[\exp(-g)]^k,$$

$$F_{11} = kg[\exp(-g)]^k[tkg - t + 1], \quad and \quad F_{02} = kx^2g[\exp(-g)]^k[kg - 1].$$

The derivatives of F are all evaluated at $x = l_\beta(\lambda)$ with $\lambda = \hat{\lambda}_n$

$$E\left[F\left(l_\beta\left(\hat{\lambda}_n\right);\lambda\right] \approx \beta + \frac{o.5tkg[\exp(-g)]^k\sigma^2\left(\hat{\lambda}_n\right)(t - tkg - 2)}{\lambda^2}, \quad (9)$$

where, $t = \lambda x = -\log\left[-\frac{1}{k}\log(\beta)\right]$ and $g = e^{-t}$. Instead of the actual value of $\sigma^2\left(\hat{\lambda}_n\right)$ we use its estimate.

4. SIMULATION STUDY –

The expectations of the β- expectation TIs are studied by using simulation. Balkrishnan and Sandhu (1995) presented algorithm for generating samples from progressively Type-II censored scheme. Use this algorithm to generate samples from lifetime distribution of k-unit parallel system where each component has Gumbel distribution.

Algorithm –

1. Generate i.i.d. observations ($W_1, W_2, \ldots, W_m$) from U(0, 1).
2. For ($R_1, R_2, \ldots, R_m$) censoring scheme, set $E_i=1/(i+R_m+R_{m-1}++R_{m-i+1})$ for i=1,2,..,m.
3. Set $V_i= W_i^{E_i}$ for i=1,2, ….. , m.
4. Set $U_i= 1-(V_m V_{m-1} \ldots V_{m-i+1})$ for i=1,2, ….. , m. Then ($U_1, U_2, \ldots, U_m$) is the progressively Type-II censored sample from U(0, 1).
5. For given values of the parameter λ, set $x_{(i)} = -\frac{1}{\lambda}\log\left[-\frac{1}{k}\log(U_i)\right]$ for i=1,2,…, m.

Then ($x_{(1)}, x_{(2)}, \ldots, x_{(m)}$) is the required progressively Type-II censored sample from the distribution of a k-unit parallel system with Gumbel distribution as the component life distribution. For simulation study, we consider 40 different progressively Type-II censored schemes. The simulation study is performed with the following.

- In Table 1-2, scheme (a, b) stands for R_1= a and R_2=b. Similar meaning holds for schemes described through completely specified vector, while scheme (10, 4*0) means R_1=10 and rest four R_i's are zero. i.e. $R_2=R_3=R_4=R_5=0$.
- The simulation was carried out for 3-unit and 5-unit parallel system (i.e. k=3, k=5) with λ=0.5. Newton-Raphson method is used to compute MLE.
- By replacing λ and $\sigma^2(\hat{\lambda})$ by their respective estimates $\hat{\lambda}$ and $\hat{\sigma}^2(\hat{\lambda})$, the expectations of the approximate β–expectation TIs were calculated and compared with the simulation results.
- For each particular progressive censoring scheme, 10,000 sets of observations were generated. The simulated mean coverage and the estimated expectation of the TI are given in Tables 1 and 2 for k=3 and k=5 respectively.

Table 1: Simulated mean and estimated expectation of the approximate β– expectation tolerance interval for k=3 and λ=0.5

n	m	Scheme No.	Scheme	Simulated Mean			Estimated Expectation		
				90%	95%	99%	90%	95%	99%
10	5	[1]	(5,4*0)	0.8552	0.9109	0.9685	0.9150	0.9675	0.9998
		[2]	(0,5,3*0)	0.8538	0.9100	0.9681	0.9150	0.9675	0.9998
		[3]	(4*0,5)	0.8517	0.9085	0.9672	0.9130	0.9651	0.9985
		[4]	(5*1)	0.8533	0.9098	0.9682	0.9136	0.9659	0.9989
	10	[5]	(10*0)	0.8783	0.9316	0.9810	0.9075	0.9587	0.9949
15	5	[6]	(10, 4*0)	0.8529	0.9089	0.9674	0.9151	0.9675	0.9998
		[7]	(4*0, 10)	0.8441	0.9020	0.9630	0.9136	0.9658	0.9989
		[8]	(2,2,2,2,2)	0.8515	0.9084	0.9674	0.9136	0.9659	0.9989
	10	[9]	(5,9*0)	0.8770	0.9308	0.9807	0.9075	0.9587	0.9949
		[10]	(9*0,5)	0.8775	0.9313	0.9811	0.9066	0.9577	0.9943
		[11]	(3,2, 8*0)	0.8750	0.9293	0.9801	0.9075	0.9588	0.9949
	15	[12]	(15*0)	0.8848	0.9373	0.9841	0.9050	0.9558	0.9933
20	10	[13]	(10, 9*0)	0.8779	0.9314	0.9810	0.9075	0.9587	0.9949
		[14]	(9*0,10)	0.8754	0.9299	0.9806	0.9065	0.9576	0.9943
		[15]	(5,5,8*0)	0.8762	0.9302	0.9805	0.9076	0.9588	0.9949
	20	[16]	(20*0)	0.8894	0.9412	0.9860	0.9038	0.9544	0.9924
25	10	[17]	(15,9*0)	0.8760	0.9301	0.9804	0.9075	0.9587	0.9949
		[18]	(9*0,15)	0.8743	0.9290	0.9801	0.9067	0.9578	0.9943
		[19]	(5,5,5,7*0)	0.8777	0.9311	0.9808	0.9076	0.9589	0.9950
	15	[20]	(10, 14*0)	0.8848	0.9375	0.9843	0.9050	0.9558	0.9933
		[21]	(14*0,10)	0.8842	0.9374	0.9844	0.9044	0.9551	0.9929
		[22]	(5,5,13*0)	0.8843	0.9371	0.9841	0.9050	0.9559	0.9934
	25	[23]	(25*0)	0.8910	0.9426	0.9867	0.9030	0.9535	0.9920
30	10	[24]	(20, 9*0)	0.8774	0.9311	0.9809	0.9074	0.9587	0.9949
		[25]	(9*0,20)	0.8715	0.9268	0.9789	0.9068	0.9580	0.9945
		[26]	(4*5,6*0)	0.8751	0.9293	0.9800	0.9077	0.9589	0.9950
	15	[27]	(15, 14*0)	0.8847	0.9373	0.9841	0.9050	0.9558	0.9932
		[28]	(14*0,15)	0.8836	0.9368	0.9841	0.9044	0.9551	0.9929
		[29]	(5,5,5,12*0)	0.8839	0.9368	0.9840	0.9050	0.9559	0.9933
	20	[30]	(10, 19*0)	0.8874	0.9399	0.9856	0.9037	0.9544	0.9924
		[31]	(19*0,10)	0.8886	0.9408	0.9860	0.9033	0.9539	0.9922
		[32]	(0,5,5,17*0)	0.8882	0.9404	0.9857	0.9038	0.9544	0.9925
	30	[33]	(30*0)	0.8926	0.9440	0.9873	0.9025	0.9529	0.9916
50	20	[34]	(30,19*0)	0.8886	0.9407	0.9858	0.9037	0.9543	0.9924
		[35]	(19*0,30)	0.8881	0.9405	0.9858	0.9033	0.9539	0.9922
		[36]	(6*5,14*0)	0.8878	0.9401	0.9856	0.9038	0.9544	0.9925
	35	[37]	(15,34*0)	0.8932	0.9445	0.9876	0.9021	0.9525	0.9914
		[38]	(34*0,15)	0.8934	0.9448	0.9878	0.9019	0.9522	0.9913
		[39]	(5,5,5,32*0)	0.8926	0.9441	0.9875	0.9021	0.9525	0.9914
	50	[40]	(50*0)	0.8950	0.9460	0.9883	0.9015	0.9518	0.9910

Table 2: Simulated mean and estimated expectation of the approximate β– expectation tolerance interval for k=5 and λ=0.5

n	m	Scheme No.	Scheme	Simulated Mean			Estimated Expectation		
				90%	95%	99%	90%	95%	99%
10	5	[1]	(5,4*0)	0.8609	0.9174	0.9739	0.9168	0.9665	0.9981
		[2]	(0,5,3*0)	0.8602	0.9173	0.9741	0.9160	0.9657	0.9977
		[3]	(4*0,5)	0.8649	0.9215	0.9765	0.9130	0.9628	0.9963
		[4]	(5*1)	0.8612	0.9184	0.9750	0.9141	0.9639	0.9968
	10	[5]	(10*0)	0.8796	0.9338	0.9829	0.9086	0.9585	0.9942
15	5	[6]	(10, 4*0)	0.8601	0.9168	0.9735	0.9170	0.9667	0.9982
		[7]	(4*0, 10)	0.8609	0.9186	0.9752	0.9122	0.9620	0.9959
		[8]	(2,2,2,2,2)	0.8619	0.9193	0.9754	0.9132	0.9630	0.9964
	10	[9]	(5,9*0)	0.8795	0.9336	0.9827	0.9086	0.9585	0.9941
		[10]	(9*0,5)	0.8819	0.9357	0.9838	0.9071	0.9570	0.9934
		[11]	(3,2, 8*0)	0.8797	0.9337	0.9828	0.9085	0.9584	0.9941
	15	[12]	(15*0)	0.8868	0.9395	0.9855	0.9058	0.9557	0.9928
20	10	[13]	(10, 9*0)	0.8801	0.9341	0.9830	0.9086	0.9585	0.9942
		[14]	(9*0,10)	0.8824	0.9361	0.9840	0.9065	0.9564	0.9931
		[15]	(5,5,8*0)	0.8798	0.9339	0.9829	0.9085	0.9583	0.9941
	20	[16]	(20*0)	0.8904	0.9424	0.9868	0.9043	0.9543	0.9921
25	10	[17]	(15,9*0)	0.8793	0.9334	0.9826	0.9087	0.9585	0.9942
		[18]	(9*0,15)	0.8821	0.9360	0.9841	0.9063	0.9562	0.9930
		[19]	(5,5,5,7*0)	0.8802	0.9342	0.9830	0.9083	0.9582	0.9940
	15	[20]	(10, 14*0)	0.8859	0.9389	0.9853	0.9058	0.9557	0.9928
		[21]	(14*0,10)	0.8872	0.9402	0.9860	0.9046	0.9545	0.9922
		[22]	(5,5,13*0)	0.8873	0.9398	0.9856	0.9057	0.9556	0.9928
	25	[23]	(25*0)	0.8926	0.9441	0.9875	0.9035	0.9534	0.9917
30	10	[24]	(20, 9*0)	0.8800	0.9338	0.9827	0.9087	0.9585	0.9942
		[25]	(9*0,20)	0.8799	0.9346	0.9836	0.9061	0.9560	0.9929
		[26]	(4*5,6*0)	0.8800	0.9341	0.9830	0.9081	0.9580	0.9939
	15	[27]	(15, 14*0)	0.8869	0.9396	0.9855	0.9058	0.9557	0.9928
		[28]	(14*0,15)	0.8890	0.9414	0.9864	0.9044	0.9543	0.9921
		[29]	(5,5,5,12*0)	0.8875	0.9400	0.9857	0.9057	0.9556	0.9927
	20	[30]	(10, 19*0)	0.8904	0.9423	0.9868	0.9043	0.9543	0.9921
		[31]	(19*0,10)	0.8919	0.9436	0.9874	0.9036	0.9535	0.9917
		[32]	(0,5,5,17*0)	0.8901	0.9423	0.9868	0.9043	0.9542	0.9921
	30	[33]	(30*0)	0.8927	0.9444	0.9877	0.9029	0.9528	0.9914
50	20	[34]	(30,19*0)	0.8901	0.9421	0.9866	0.9044	0.9543	0.9921
		[35]	(19*0,30)	0.8909	0.9431	0.9873	0.9031	0.9531	0.9915
		[36]	(6*5,14*0)	0.8899	0.9421	0.9867	0.9042	0.9541	0.9920
	35	[37]	(15,34*0)	0.8948	0.9458	0.9883	0.9025	0.9524	0.9912
		[38]	(34*0,15)	0.8950	0.9461	0.9884	0.9021	0.9520	0.9910
		[39]	(5,5,5,32*0)	0.8947	0.9458	0.9883	0.9025	0.9524	0.9912
	50	[40]	(50*0)	0.8959	0.9468	0.9887	0.9017	0.9517	0.9908

5. CONCLUSIONS –

Results of simulation study shows that simulated mean of the coverage increase as sample size *n* and effective sample size *m* increase. The estimated expectation of the coverage of the approximate expectation TI shows satisfactory results for large *n*. Simulated mean and estimated expectation of the coverage increase as number of units *k* in parallel system increases. For *k*=3, simulated mean have better coverage for progressive censoring scheme, whereas for *k*=5, simulated mean have better coverage for conventional censoring scheme, for small sample size. Overall both conventional Type-II censoring scheme and progressive Type-II censoring scheme gives better result.

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