



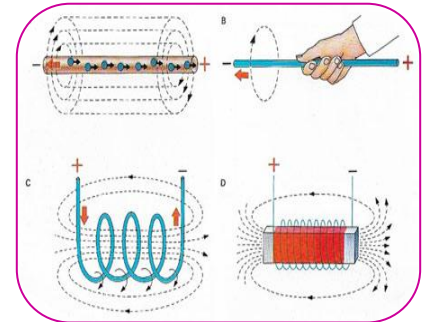
## BIANCHI TYPE VI<sub>0</sub> UNIVERSE FILLED WITH ELECTROMAGNETIC FIELD IN $f(R)$ THEORY OF GRAVITATION

V. M. Raut

Department of Mathematics, Shri Shivaji Science College, Morshi Road, Amravati .  
E mail – vinayraut18@gmail.com

### ABSTRACT:

In the  $f(R)$  theory of gravity we have studied electromagnetic field in Bianchi type-VI<sub>0</sub> space time by considering the general case. It is found that if the study is confined to the case of  $f(R) = \eta R$ . We consider the  $f(R)$  modified theory of gravity, where the Lagrangian is given by an arbitrary function of the Ricci scalar  $R$  and of the trace of the stress-energy  $T$ . In this paper we have discussed solution of Maxwell's equations and the components of the vector potentials. Some physical and kinematical behaviours of the model are also discussed.



**KEYWORD:** Bianchi Type -VI<sub>0</sub>, Electromagnetic field,  $f(R)$  theory of gravity, isotropy, constant vector potential.

### 1. INTRODUCTION:

In the modified theory of gravity, now a days there has lot of interest of cosmologists in the view of the direct evidence of late time accelerated expansion of the universe which comes from high redshift supernova experiment Riessel *et al.*,(2004). There are mainly two approaches in  $f(R)$  theory of gravity. The first is called “metric approaches” in which the connection is the Levi-Civita connection and the variation of the action is done with respect to metric. The second approach is “Platini formalism” in which connection and the metric are considered independent of each other and the variation done for parameters independently.

The idea of introducing additional terms of the Ricci scalar to the Einstein-Hilbert action did not begin years ago with the  $f(R)$  theory of gravity by Carroll *et al.*,(2004). Sharif and Shamir,(2011) have studied plane symmetric solution in  $f(R)$  gravity.

The  $f(R)$  theory of gravitation formulated by Nojiri and Odintsov, (2007). There are two kinds of alternative accelerated expansion of the universe have been proposed for this unexpected observational phenomenon. One is negative pressure known as dark energy (DE), which induces a late-time accelerating cosmic expansion. The other one is the modified gravity, which originates from the idea that the general relativity is inadequate in the cosmic scale and therefore need to be modified.

In order to explain the nature of the DE and accelerated expansion, a variety of theoretical models have been proposed in literature. In our opinion, one of interesting and prospective version of

modified gravity theories is the  $f(R, T)$  gravity proposed by Harko *et. al.*, (2010). The exact solutions of  $f(R, T)$  field equations for locally rotationally symmetric Bianchi type-I cosmological model discussed by Adhav, (2012). Mete and Mule, (2017) have studied Bianchi-VI<sub>0</sub> magnetized cosmological model in  $f(R, T)$  gravity. Bijan Saha, (2015) has studied the interacting scalar and electromagnetic fields in Bianchi type-I universe. Solanke and Karade, (2016) have studied plane symmetric universe filled with combination of perfect fluid and scalar field with electromagnetic fields in  $f(R, T)$ .

Samanta C.G, (2013) has studied universe filled with dark energy from a wet dark fluid in  $f(R, T)$  theory of gravity. Many authors have investigated different problem within the scope of  $f(R, T)$  theory.

Magnetic field plays a vital role in description of energy distribution in the universe as it contains highly ionized matter. Strong magnetic fields can be created due to adiabatic compression in cluster of galaxies. The presence of magnetic fields in galactic and intergalactic spaces is evident from recent observations by Grasso and Rubinstein (2001). The large scale magnetic field can be detected by observing their effects on the CMB radiation. These fields would enhance anisotropies in the CMB, since the expansion rate will be different depending on the direction of field lines by Madson (1989). Melvin (1975) suggested that the presence of magnetic field is not unrealistic as it appears to be because during the evolution of the universe, matter was in highly ionized state, smoothly coupled with the field subsequently form neutral matter due to universe expansion. Our interest is to explore the role of electromagnetic field played in the amended  $f(R)$  gravity in Bianchi type- VI<sub>0</sub> metric universe.

**2. THE METRIC AND FIELD EQUATIONS:**

We consider the Bianchi type-VI<sub>0</sub> universe specified in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{-2m^2 x} dy^2 - C^2 e^{2m^2 x} dz^2, \tag{2.1}$$

where  $A, B$  and  $C$  are functions of time  $t$ .

The field equation of  $f(R, T)$ , theory (Harko *et. al.*, 2010) are deduced by varying the action

$$S = \int f(R, T) \sqrt{-g} d^4 x + \int L_m \sqrt{-g} d^4 x, \tag{2.2}$$

where  $f(R, T)$  is an arbitrary function of Ricci scalar  $R, T$  is the trace of the stress energy matter and  $L_m$  is the matter of Lagrangian.

Varying the action (2.2) with respect to  $g^{ij}$  which yields as

$$\delta S = \frac{1}{2x} \left\{ f_R(R, T) \frac{\partial R}{\partial g^{ij}} + f_T(R, T) \frac{\partial T}{\partial g^{ij}} + \frac{f(R, T)}{\sqrt{-g}} \frac{\partial \sqrt{-g}}{\partial g^{ij}} + \frac{2\chi}{\sqrt{-g}} \frac{\partial (L_m \sqrt{-g})}{\partial g^{ij}} \right\} \sqrt{-g} d^4 x \tag{2.3}$$

Considering  $\delta S = 0$  and using equation (2.3), we obtain

$$f_R(R, T) R_{ij} - \frac{1}{2} f(R, T) g_{ij} + (g_{ij} \square - \nabla_i \nabla_j) f_R(R, T) = \chi T_{ij} - f_T(R, T) [T_{ij} + \theta_{ij}], \tag{2.4}$$

where  $f_R(R, T) = \frac{\partial f(R)}{\partial R}$ ,  $f_T(R, T) = \frac{\partial f(R)}{\partial T}$ ,  $\theta_{ij} = g^{\alpha\beta} \frac{\partial T_{\alpha\beta}}{\partial g^{ij}}$  and  $\nabla_i$  is the covariant derivative.

Replacing  $f(R, T)$  by  $f(R)$  in (2.4), we obtain

$$f_R(R)R_{ij} - \frac{1}{2}f(R)g_{ij} + (g_{ij}\square - \nabla_i\nabla_j)f_R(R) = \chi T_{ij}. \tag{2.5}$$

Taking trace of equation (2.5), we get

$$\square f_R(R) = \frac{1}{3}\chi T + \frac{2}{3}f(R) - \frac{1}{3}f_R(R). \tag{2.6}$$

Let us consider the particular case  $f(R) = \eta R$ .

It follows notation

$$f(R) = \frac{\partial f(R)}{\partial R} = \frac{\partial}{\partial R} \eta R = \eta. \tag{2.7}$$

The field equation (2.5) with an aid of equation (2.7), yields

$$\eta R_{ij} - \frac{1}{2}\eta R g_{ij} = \chi T_{ij}. \tag{2.8}$$

Using equation (2.7), equation (2.6) yields

$$\chi T + (\eta R) = 0. \tag{2.9}$$

Using equations (2.8) and (2.9), we obtained

$$\eta R_{ij} + \frac{1}{2}(\chi T)g_{ij} = \chi T_{ij}. \tag{2.10}$$

**[3] Energy Momentum Tensor with Electromagnetic Field Tensor:**

Energy momentum tensor for electromagnetic field is given by

$$T_{ij} = L_m g_{ij} - 2 \frac{\partial L_m}{\partial g^{ij}}, \tag{3.1}$$

where  $L_m = \frac{1}{4} F_{kl} F^{kl}$  is an the matter of Lagrangian

and  $F_{kl}$  electromagnetic field. (3.2)

$$\frac{\partial L_m}{\partial g^{ij}} = \frac{1}{2} g^{ak} F_{ai} F_{kj}. \quad (3.3)$$

Using equations(3.3), (3.2), the equation (3.1), reduces to

$$T_{ij} = \frac{1}{4} F_{kl} F^{kl} g_{ij} - F_{ki} F_j^k. \quad (3.4)$$

The electromagnetic field tensor is given by

$$F_{ij} = \frac{\partial V_i}{\partial x^j} - \frac{\partial V_j}{\partial x^i}. \quad (3.5)$$

To achieve the capability with non-static space time (2.1), we assume electromagnetic vector potential in the form

$$V_i = [u(\alpha)v_1(t), v_2(t), v_3(t), v_4(t)]. \quad (3.6)$$

From equations (3.5) and (3.6), we can easily deduce

$$F_{14} = u\dot{v}_1, F_{24} = \dot{v}_2, F_{34} = \dot{v}_3, F_{41} = -u\dot{v}_1, \quad (3.7)$$

$$F^{14} = -\frac{u\dot{v}_1}{A^2}, F^{24} = -\frac{\dot{v}_2}{B^2 e^{-2m^2x}}, F^{34} = -\frac{\dot{v}_3}{C^2 e^{2m^2x}}, F^{41} = \frac{u\dot{v}_1}{A^2}, \quad (3.8)$$

Using equations (3.7) and (3.8), we compute

$$F_\mu F^\mu = -2 \left[ \frac{u\dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{C^2 e^{2m^2x}} \right]. \quad (3.9)$$

Using equation (3.4), we establish the following nonzero components of the energy momentum tensor of material field

$$T_1^1 = \frac{1}{2} \left[ \frac{u\dot{v}_1^2}{A^2} - \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} - \frac{\dot{v}_3^2}{C^2 e^{2m^2x}} \right], \quad (3.10)$$

$$T_2^2 = \frac{1}{2} \left[ \frac{u\dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} - \frac{\dot{v}_3^2}{C^2 e^{2m^2x}} \right], \quad (3.11)$$

$$T_3^3 = \frac{1}{2} \left[ -\frac{u^2 \dot{v}_1^2}{A^2} - \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{C^2 e^{-2m^2x}} \right], \tag{3.12}$$

$$T_4^4 = \frac{1}{2} \left[ -\frac{u \dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{C^2 e^{-2m^2x}} \right] \tag{3.13}$$

From equations (3.10) to (3.13), we deduced the components of energy tensor as

$$T_j^i = 0, \text{ for } i \neq j. \tag{3.14}$$

**4. MAXWELL’S EQUATIONS AND VECTOR POTENTIALS:**

The Maxwell’s Equations (Meteeet. al, 2017) are given by

$$\frac{\partial}{\partial x^j} [\sqrt{-g} F^{ij}] = 0. \tag{4.1}$$

Using equations (3.7) and (3.8), above equation gives

$$\frac{(\dot{v}_1)^\bullet}{v_1} + \frac{\dot{v}_1^2}{v_1^2} + \frac{\dot{v}_1}{v_1} \left[ \frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right] = 0, \tag{4.2}$$

$$\frac{(\dot{v}_2)^\bullet}{v_2} + \frac{\dot{v}_2^2}{v_2^2} + \frac{\dot{v}_2}{v_2} \left[ \frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B} \right] = 0, \tag{4.3}$$

$$\frac{(\dot{v}_3)^\bullet}{v_3} + \frac{\dot{v}_3^2}{v_3^2} + \frac{\dot{v}_3}{v_3} \left[ \frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right] = 0, \tag{4.4}$$

$$u = c, \tag{4.5}$$

where *c* is a constant

From equation (3.4), we have

$$T_2^1 = 0 = \dot{v}_1 \dot{v}_2, \quad T_3^1 = 0 = \dot{v}_1 \dot{v}_3, \quad T_3^2 = 0 = \dot{v}_2 \dot{v}_3. \tag{4.6}$$

Using equations (4.1) to (4.6), we write

$$\frac{\dot{v}_1 \dot{v}_2}{v_1 v_2} = \frac{\dot{v}_1 \dot{v}_3}{v_1 v_3} = \frac{\dot{v}_2 \dot{v}_3}{v_2 v_3} = 0, \tag{4.7}$$

which further implies

$$\frac{\dot{v}_1}{v_1} = \frac{\dot{v}_2}{v_2} = \frac{\dot{v}_3}{v_3} = \frac{\dot{D}}{D}, \tag{4.8}$$

where  $D$  is some unknown function of  $t$ .  
Using equations(4.6) and (4.7), we get

$$\left(\frac{\dot{D}}{D}\right)^2 = 0. \tag{4.9}$$

Using equation (4.7), we obtain

$$v_1 = k_1 D, v_2 = k_2 D, v_3 = k_3 D, \tag{4.10}$$

where  $k$  's are constants of integration.

**5.SOLUTION OF FIELD EQUATIONS:**

Now our plan is to express the components of  $T_j^i$  in terms of  $T_4^4$ . For this, we consider the expression as

$$\frac{u^2 \dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{a_3^2 e^{2m^2x}} = \left[ \frac{u^2 v_1^2}{A^2} \left(\frac{\dot{D}}{D}\right)^2 + \frac{v_2^2}{B^2 e^{-2m^2x}} \left(\frac{\dot{D}}{D}\right)^2 + \frac{v_3^2}{C^2 e^{2m^2x}} \left(\frac{\dot{D}}{D}\right)^2 \right]$$

$$\frac{u^2 \dot{v}_1^2}{A^2} + \frac{\dot{v}_2^2}{B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{C^2 e^{2m^2x}} = \left[ \frac{u^2 v_1^2}{A^2} + \frac{v_2^2}{B^2 e^{-2m^2x}} + \frac{v_3^2}{C^2 e^{2m^2x}} \right] \left(\frac{\dot{D}}{D}\right)^2 = -I \left(\frac{\dot{D}}{D}\right)^2$$

$$T_4^4 = \frac{u^2 \dot{v}_1^2}{2A^2} + \frac{\dot{v}_2^2}{2B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{2C^2 e^{2m^2x}} = -\frac{1}{2} I \left(\frac{\dot{D}}{D}\right)^2 \tag{5.1}$$

$$T_1^1 = \frac{u^2 \dot{v}_1^2}{2A^2} - \frac{\dot{v}_2^2}{2B^2 e^{-2m^2x}} - \frac{\dot{v}_3^2}{2C^2 e^{2m^2x}} = -T_4^4 + \frac{u^2 v_1^2}{C^2} \left(\frac{\dot{D}}{D}\right)^2, \tag{5.2}$$

$$T_2^2 = -\frac{u^2 \dot{v}_1^2}{2A^2} + \frac{\dot{v}_2^2}{2B^2 e^{-2m^2x}} - \frac{\dot{v}_3^2}{2C^2 e^{2m^2x}} = -T_4^4 + \frac{v_2^2}{B^2 e^{-2m^2x}} \left(\frac{\dot{D}}{D}\right)^2, \tag{5.3}$$

$$T_3^3 = -\frac{u^2 \dot{v}_1^2}{2A^2} - \frac{\dot{v}_2^2}{2B^2 e^{-2m^2x}} + \frac{\dot{v}_3^2}{2C^2 e^{2m^2x}} = -T_4^4 + \frac{v_3^2}{C^2 e^{2m^2x}} \left(\frac{\dot{D}}{D}\right)^2. \tag{5.4}$$

Using equations (5.1) to (5.4), we get trace of energy momentum tensor as

$$T = I\left(\frac{\dot{D}}{D}\right)^2 - I\left(\frac{\dot{D}}{D}\right)^2 = 0. \tag{5.5}$$

Considering the non-trivial components of Ricci tensor from field equation (2.10), we get

$$\eta\left[-\frac{2m^4}{A^2} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC}\right] = \chi\left[-T_4^4 + \frac{u^2v_1^2}{A^2}\left(\frac{\dot{D}}{D}\right)^2\right], \tag{5.6}$$

$$\eta\left[\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC}\right] = \chi\left[-T_4^4 + \frac{u^2v_1^2}{B^2}\left(\frac{\dot{D}}{D}\right)^2\right], \tag{5.7}$$

$$\eta\left[\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC}\right] = \chi\left[-T_4^4 + \frac{u^2v_1^2}{C^2}\left(\frac{\dot{D}}{D}\right)^2\right], \tag{5.8}$$

Using equations (4.9) and (5.6) to (5.8), we have

$$-\frac{2m^4}{A^2} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} = 0, \tag{5.9}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} = 0, \tag{5.10}$$

$$\frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} = 0. \tag{5.11}$$

With the help of equation (4.8), we can write the equations (4.2) to (4.4) as

$$\left(\frac{\dot{D}}{D}\right)^{\bullet} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left[\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - \frac{\dot{A}}{A}\right] = 0, \tag{5.12}$$

$$\left(\frac{\dot{D}}{D}\right)^{\bullet} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left[\frac{\dot{A}}{A} + \frac{\dot{C}}{C} - \frac{\dot{B}}{B}\right] = 0, \tag{5.13}$$

$$\left(\frac{\dot{D}}{D}\right)^{\bullet} + \left(\frac{\dot{D}}{D}\right)^2 + \frac{\dot{D}}{D}\left[\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right] = 0. \tag{5.14}$$

Equations(5.12) to (5.14) further imply that

$$\frac{\dot{A}}{A} = \frac{\dot{B}}{B} = \frac{\dot{C}}{C}, \tag{5.15}$$

which on integration with respect to  $t$  gives

$$A = k_4 a_2, B = k_5 a_3, C = k_6 a_1, \tag{5.16}$$

where  $k$ 's are constants of integration.  
 From equations (5.9) to (5.11) and (5.15), we deduce

$$\frac{\ddot{B}}{B} + 2\left(\frac{\dot{B}}{B}\right)^2 = 0, \tag{5.17}$$

$$\frac{\ddot{C}}{C} + 2\left(\frac{\dot{C}}{C}\right)^2 = 0. \tag{5.18}$$

On integration, we get

$$B = (3k_7 t + 3k_8)^{\frac{1}{3}}, \tag{5.19}$$

$$C = (3k_9 t + 3k_{10})^{\frac{1}{3}}. \tag{5.20}$$

With the help of equations (5.16) and (5.19), yield

$$A = (3k_{11} t + 3k_{12})^{\frac{1}{3}}. \tag{5.21}$$

Using the equations (5.15), (5.19) to (5.21), we get

$$k_7 = k_9 = k_{11} \text{ and } k_8 = k_{10} = k_{12},$$

Let  $k_7 = k_9 = k_{11} = d_1, k_8 = k_{10} = k_{12} = d_2.$

$$A = B = C = (3d_1 t + 3d_2)^{\frac{1}{3}}. \tag{5.22}$$

From equation (4.9), we get

$$\frac{\dot{D}}{D} = 0$$

Integrating, we get

$$D = C, \tag{5.23}$$



where  $C$  is a constant.

From equations (5.5.23) and (5.4.9), we have

$$v_1 = k_{13}, v_2 = k_{14}, v_3 = k_{15}, v_4 = 0.$$

From equations (5.22) and (5.9), we deduce

$$m = 0$$

Using equation (5.22), the line element (2.1), reduces to

$$ds^2 = dt^2 - (3d_1t + 3kd_2)^{\frac{2}{3}} \left[ dx^2 + e^{-2m^2x} dy^2 + e^{2m^2x} dz^2 \right], \tag{5.24}$$

where  $k$ 's are constants of integration.

**[6] The Physical Significance of the Model:**

The physical quantities of observational interest in cosmology are, the expansion scalar ( $\theta$ ), shear scalar ( $\sigma$ ), spatial volume ( $V$ ), The mean Hubble parameter ( $H$ ), the mean anisotropic parameter ( $A_m$ ), the deceleration parameter ( $q$ ) and the cosmic Jerk parameter ( $J$ ), for the model (5.24) defined as

$$\theta = 3H = \left( 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B} \right), \tag{6.1}$$

$$\sigma^2 = \frac{1}{2} \sum_{i=1}^3 H_i^2 - \frac{\theta^2}{6}, \tag{6.2}$$

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2, \tag{6.3}$$

$$q = \frac{d}{dt}(H) - 1, \tag{6.4}$$

$$J = q + 2q^2 - \frac{\dot{q}}{H}, \tag{6.5}$$

The spatial volume is obtained as

$$V = 3(d_1t + d_2). \tag{6.6}$$

The mean Hubble parameter is given by

$$H = \frac{d_1}{3(d_1 t + d_2)}. \quad (6.7)$$

The expansion scalar is obtained as

$$\theta = 3H = \frac{d_1}{(d_1 t + d_2)}. \quad (6.8)$$

The shear scalar gives

$$\sigma^2 = 0. \quad (6.9)$$

The mean anisotropic parameter  $A_m$  as

$$A_m = 0. \quad (6.10)$$

The deceleration parameter is given by

$$q = d_1 - 1. \quad (6.11)$$

This equation produces a constant value for deceleration parameter and can have both positive as well as negative values. Positive value of deceleration parameter gives the standard deceleration model while the negative value results into inflation or the accelerating universe.

The cosmic Jerk parameter deduced

$$J = 2d_1^2 - 3d_1 + 1. \quad (6.12)$$

## CONCLUSION:

In this paper, we have considered the particular case  $f(R) = \eta R$  model in Bianchi Type-VI universe. The volume of the universe is increasing with the increasing time. The mean Hubble parameter is constant for  $t \rightarrow 0$ , and null at  $t \rightarrow \infty$ . The expansion scalar  $\theta \rightarrow 0$  as  $t \rightarrow \infty$  indicates that the universe is expanding with increase of time and the rate of expansion decreases with the increase of time. The anisotropic parameter and the shear scalar found to be zero, hence the universe does not approach anisotropy and the universe is shearing free. For  $d_1 > 1$  the sign of  $q$  becomes positive which corresponds to the standard decelerating behavior of the model. As we ensure about decelerating, the model is also consistent with the recent CMB observation model by WMAP, as well as with the high redshift supernovae Ia data, whereas for  $d_1 < 1$  the sign of  $q$  becomes negative, which corresponds to the standard accelerating behavior of the model. This scenario is consistent with recent observations.

## REFERENCES:

- [1] Adhav K S, (2012),: Astrophysics. Space sci. **339**, 365
- [2] Bijan Saha, (2015),: Int. j. of Phy.. 1073-75, 31.
- [3] Carroll S.M, Duvvuri V and Turner M S, (2004),: Phys. Rev. D 70, 043528.

- 
- [4] Grasso D. Rubinstein HR,(2001).:Phys. Rep.; **348**: 163-266.
  - [5] Harko T and F S N Lobo.,(2010).: Eur. Phys. J. C 70, 373
  - [6] Madsen,M.S.(1989).:Astronomical Society,**237**,109- 117
  - [7] Melvin M.A.,(1975).: Ann. New York Acad. Sci.; **262**: 253-274.
  - [8] Mete V G and Mule K R ( 2017).:Int. j.of IJRBT,.vol.5,issue2,pp:1149-1156.
  - [9] Nojiri S, Odintsov S D,(2007): Int. J. Mod. Phys. 4 115, hep-th/0601213.
  - [10] Nojiri S, Odintsov S D., (2007).: Phys. Lett. B **651**, 22
  - [11] Riess A.,(2004).:Astron.J.607,665
  - [12] Samanta C.G .,(2013).:IntJ.Theor. Physics 1507-17013.
  - [13] Sharif. Mand Shamir Farast M.,(2011).:Modern Phy. Lett.arXiv:9012.1393.
  - [14] Solanke D.T and Karade T.M ,(2016).:Prespacetime, J.vol.7,issue12,pp:1551