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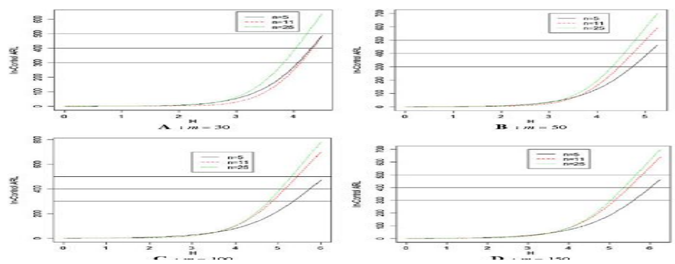

DISTRIBUTION-FREE CONTROL CHART FOR PROCESS VARIABILITY

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ABSTRACT :

In this paper, distribution-free or nonparametric control chart based on Sukhatme test statistics is developed for process variability. The proposed control chart is investigated by a simulation study under normal, Laplace and uniform distributions. The performance of the proposed chart is improved by using the run rule and compared with the normal, Laplace and uniform distributions.

KEYWORDS: *ARL, Control chart, Sukhamte test statistic. Process Variability.*

INTRODUCTION :

A definition of distribution-free or nonparametric control chart is given in terms of its in-control run length distribution. The number of samples that desires to be collected before the first out-of-control signal is given by a chart is a random variable called the run-length; the probability distribution of the run-length is referred to as the run-length distribution. If the in-control run length distribution is identical for every continuous distribution then the chart is called distribution-free or nonparametric (Chakraborti et al. (2004)).

Shewhart control charts are most popular control charts for monitor process variability. Control charts are based on the basic assumption that the underlying distribution of the excellence quality is normal. In reality this assumption may not hold in all situations. In such situation development and application do not depend on a particular distributional assumption is popular. Distribution-free or nonparametric control charts can serve this purpose. The main advantage of distribution-free chart is that it does not assume any probability distribution for the quality of interest

In literature, several nonparametric control charts are proposed for monitor location of univariate distributions. Some of these are based on sign and/or rank statistics by assuming a known in-control target value for process location. Sukhamte (1956) presented on certain two-sample nonparametric tests for variances. Bakir and Reynolds (1979) developed a nonparametric CUSUM to monitor a process center based on with-in group signed-ranks. Amin, Reynolds and Bakir (1995) developed Shewhart and cumulative sum (CUSUM) control charts based on sign test statistic. Bakir (2004) developed a distribution-free Shewhart control chart for monitoring process center based on the signed-ranks of grouped observations. Bakir (2006) presented Shewhart, CUSUM and EWMA control charts based on signed-rank-like statistics of grouped data for monitoring a process center when in-control target center was not specified and studied the robustness of the charts against outliers. Chakraborti et al. (2001) presented an extensive overview of the literature on univariate nonparametric control charts. Das (2008) presented the efficiency of control charts for monitoring variability based on two-sample nonparametric tests. Zombade and Ghute developed (2014) nonparametric control charts for variability using runs rules

Many nonparametric tests like sign, signed-rank has been proposed in the literature. In this paper, we use the Sukhatme test statistic as a chart statistic for detect process variability. The proposed nonparametric chart for monitor the process variability is based on two independent samples drawn from two different distributions. The rest of the paper is organized as follows. A brief introduction of the proposed chart given in Section 2. The performance of proposed chart is evaluated and compared in Section 3. Some conclusions are given in Section 4.

2. NONPARAMETRIC SUKHATME TEST STATISTIC

Suppose $X = (X_1, X_2, ..., X_m)$ and $Y = (Y_1, Y_2, ..., Y_n)$ be two independent samples drawn from two different distributions. If both the X's and Y's are comes from normal distribution, which are commonly used in statistical tests for comparing dispersions. We will assume that both distributions are absolutely continuous and they are different in the scale parameters.

Definition: The control chart statistic based on Sukhamte test:

$$T = \sum_{i=1}^{m} \sum_{j=1}^{n} D_{ij} \tag{1}$$

Where $D_{ij} = 1$, if $0<X<Y$ or $Y<X<0$

$= 0$, otherwise

The mean and variance of the statistic T is given by,

$$E(T) = \frac{1}{4} \tag{2}$$

$$Var(T) = \frac{(m+n+7)}{48mn} \tag{3}$$

The chart signals an out-of-control if $T < LCL$ or $T > UCL$, where LCL and UCL denote the lower and upper control limits, respectively. The upper control limit UCL = 3 and LCL = -3. The process is considered out-of-control when a plotted point lies above UCL or below LCL.

$$Z = \frac{T - E(T)}{\sqrt{Var(T}} \tag{4}$$

The sample statistics Z computed from independent observations from the process are plotted against an upper control limit UCL = 3 and LCL = -3. The process is considered out-of-control when a plotted point lies above UCL or below LCL.

3. Performance of the Proposed Chart

A proper measure of the chart performance is the expected value of the run length of control chart called the average run length (ARL). The run length is the number of samples or sub-groups that need to be collected before the first out of control signal is called the run length. It necessary that the ARL values of a chart be large when the process is in control. If the values of the in-control ARL are larger, then the performance of the chart is better with respect to false alarms. The event when a chart signals an out-of-control situation is called a signal event. When a chart give signals but the process is actually in control the signal event, is called false alarm. The main purpose of a control chart is to detect the change in the process as early as possible and give an out-of-control signal. Clearly the chart gives the quick detection and then signal, the chart is more capable.

Computer programs written in C are used to study the performance of the proposed control chart under normal, uniform and Laplace distributions. Simulation by 10000 runs is used for sample size of n = 10, 15 and 20. The three distributions are selected, since they are symmetric and value of standard deviation (dispersion) σ can be changed without changing the mean or median.

Examinations of Table 1 to Table 3, provide the ARL values of the proposed charts when the underlying process follows normal, Laplace and uniform distributions with sample sizes *n* =10, 15 and 20 respectively. In-control ARL values of the proposed chart under normal, Laplace and uniform process distributions. Out-of-control ARL values of proposed chart are smaller under uniform than normal and Laplace process distribution for sample sizes n=10, 15, and 20.

Table 1 to Table 3 lead to the following findings:

- In-control ARL values of the proposed control charts for different process distributions are approximately same. Out-of-control ARL values of proposed chart are smaller under uniform than normal and Laplace distributions. Therefore, proposed chart is more efficient under uniform than normal and Laplace distributions of the sample sizes n=10, 15 and 20 respectively.

Table No.1

ARL values of Sukhamte chart under Normal, Laplace and Uniform of sizes, n=10.

Shift	Sukhamte Chart		
σ	Normal	Laplace	Uniform
1.0	332.45	340.16	339.58
1.2	158.54	189.13	116.89
1.4	85.85	133.77	53.03
1.6	51.65	75.38	29.67
1.8	34.50	52.40	19.90
2.0	24.85	40.09	14.01
2.2	18.59	30.24	10.71
2.4	15.00	24.80	8.70
2.6	12.24	20.71	7.44
2.8	10.04	17.16	6.38
3.0	8.84	14.98	5.60

Table No.2

ARL values of Sukhamte charts under Normal, Laplace and Uniform of sizes, n=15

Shift σ	Sukhatme Chart		
	Normal	Laplace	Uniform
1.0	414.70	419.74	415.25
1.2	164.85	213.42	105.95
1.4	68.36	103.25	35.08
1.6	34.13	58.25	16.18
1.8	19.71	34.98	9.46
2.0	12.61	23.20	6.41
2.2	8.94	16.49	4.80
2.4	6.56	12.65	3.76
2.6	5.29	9.76	3.14
2.8	4.30	8.03	2.69
3.0	3.72	6.65	2.41

Table No.3

ARL values of Sukhatme chart under Normal, Laplace and Uniform of sizes, n=20.

Shift σ	Sukhatme Chart		
	Normal	Laplace	Uniform
1.0	332.45	340.16	339.58
1.2	158.54	189.13	116.89
1.4	85.85	133.77	53.03
1.6	51.65	75.38	29.67
1.8	34.50	52.40	19.90
2.0	24.85	40.09	14.01
2.2	18.59	30.24	10.71
2.4	15.00	24.80	8.70
2.6	12.24	20.71	7.44
2.8	10.04	17.16	6.38
3.0	8.84	14.98	5.60

4. CONCLUSION

Simulation study indicates that the performance of the proposed chart is compared with the normal, Laplace and uniform distributions. The proposed chart is more efficient under uniform distribution of the sample sizes n= 10, 15 and 20 respectively.

The Sukhatme chart is more efficient under uniform distribution than the normal and Laplace distributions.

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