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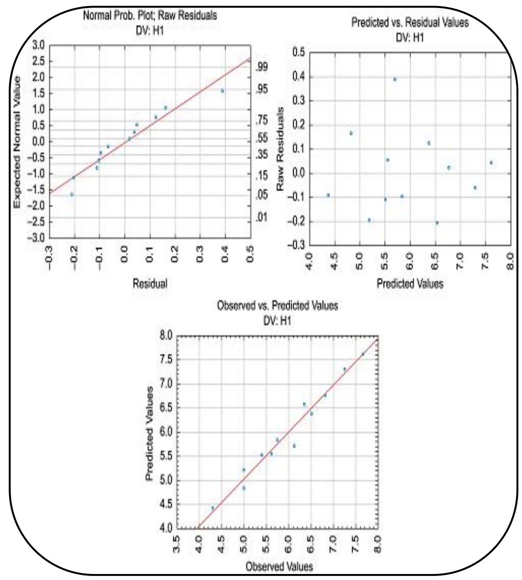
Review Of Research

NONPARAMETRIC SHEWHART-TYPE (SP) CHART FOR JOINT MONITORING OF LOCATION AND SCALE

D. M. Zombade
Department of Statistics ,
Walchand College of Arts and Science,
Solapur (MS), India.



ABSTRACT

Now a day due to automation in manufacturing process the assumptions of parametric control charts are violated which is strongly affecting the performance of control charts. In this paper, a single nonparametric Shewhart-type control chart is developed for joint monitoring of location and scale parameters of a continuous process distribution. The proposed chart is based on two nonparametric sample Lepage-type test statistic. The test statistic combines the Wilcoxon statistic and Mood statistic for jointly detecting location and scale changes respectively. The in-control and out-of-control performance of the proposed control chart is evaluated through average run length (ARL) for the normal, double exponential and uniform distributions.

KEY WORDS: *ARL, SP chart, Wilcoxon statistic, Mood statistic Control chart.*

1. INTRODUCTION

Most of the parametric control charts for joint monitoring the mean and variability of a process are based on the assumption that process distribution is normal. However, in many applications there is not always enough knowledge or information to support the assumption that process distribution is of specific shape or form such as normal. In such cases nonparametric control charts can be useful.

In joint monitoring nonparametric control charts one for monitoring the process location and another for monitoring the scale parameter of a process. Joint monitoring of a process involves two parameters, the mean (location) and variance (scale) and usually uses an efficient statistic for monitoring each parameter. Using two separate charts for monitoring mean and standard deviation can sometimes be complicated in practice for the interpretation of signals because the effect of changes in one of the parameters can affect the changes in other one. The joint monitoring scheme with single chart has received more awareness in the recent literature due to simplicity and clarity. Nonparametric joint monitoring scheme is an important area for research and literature in this area is currently very limited and thus presents a great opportunity for further research. The purpose of this paper is to contribute the research on joint monitoring scheme.

2. REVIEW OF LITERATURE

Some control charts are currently available for jointly monitoring the mean and variance which are focused on parametric control chart. Gan (1997) developed a single chart for the joint monitoring of the mean and variance of normally distributed process when the process mean and variance are both known. Chen and Cheng (1998) presented a Max chart which combines two normalized statistics, one for the mean one for the variance, by taking the maximum of the absolute values of two statistics. Chen et al. (2001) proposed MaxEWMA chart as a combination of two EWMA charts into one and they showed that their chart is effective in detecting both increase and decrease in the mean and/or the variance. Costa and Rahim (2004) developed a single noncentral Chi-square chart for joint monitoring of mean and variance. Khoo and Yap (2005) developed a single moving average control chart for joint monitoring of process mean and variability. The chart combines the usual $\overline{X}$ chart and S chart into a single chart. The combined chart is based on moving average statistics. Cheng and Thaga (2006) provided a review of literature on joint monitoring of control charts up to 2005. Recently, McCracken and Chakraborti (2013) presented an overview of literature on joint monitoring control charts. They also discussed some of the joint monitoring schemes for multivariate processes, autocorrelated data, and individual observations. Most of the parametric control charts for joint monitoring the mean and variability of a process are based on the assumption that process distribution is normal. However, in many applications there is not always enough knowledge or information to support the assumption that process distribution is of specific shape or form such as normal. In such cases nonparametric control charts can be useful. The literature in the area of nonparametric joint monitoring schemes is currently very limited. A few nonparametric joint monitoring schemes are available in the literature. Zou and Tsung (2010) proposed EWMA control chart based on goodness-of-fit test. It has been shown that the proposed chart is effective for detecting changes in location, scale and shape. Mukherjee and Chakraborti (2012) proposed a single distribution-free control chart for joint monitoring of location and scale. The chart is based on nonparametric test for location-scale by Lepage (1971) which combines the Wilcoxon rank sum (WRS) location statistic and with Ansari-Bradely scale statistic. The purpose of this paper is to contribute the research on joint monitoring scheme. In this paper, a single nonparametric Shewhart-type control chart is developed for joint monitoring of location and scale parameters of a continuous process distribution. The proposed chart is based on nonparametric two sample Lepage-type test developed by Pettitt (1976). The test combines the Wilcoxon statistic and Mood statistic for jointly detecting location and scale changes. The in-control and out-of-control performance of the proposed control chart is evaluated through average run length for the normal, double exponential and uniform distributions.

3. NONPARAMETRIC TESTS FOR LOCATION AND SCALE

Testing of hypotheses is one of the most important problems in performing nonparametric statistics. In testing of hypothesis problems, we have to test the location parameter, the scale parameter and jointly location-scale parameters are briefly discussed as follow.

Pettitt Test for Location and Scale

After Lepage statistic was proposed, various Lepage-type statistics have been proposed and discussed by many authors in the literature. One of the most famous and powerful Lepage-type statistic is a combination of the Wilcoxon and Mood (1955) statistic proposed by Pettitt (1976). The Pettitt test statistic is given as

$$T = \left(\frac{W - E(W)}{\sqrt{Var(W)}}\right)^2 + \left(\frac{M - E(M)}{\sqrt{Var(M)}}\right)^2 \quad (1)$$

Where W is Wilcoxon rank sum statistic for location shift and M is the Mood rank statistic for scale shift. To adopt the idea of two sample test for control chart implementation, *m* independent observations from an in-control process are used as reference sample and compared to future sample subgroups of *n* independent observations.

Shewhart- Pettitt Chart

In this Section, we develop a nonparametric control chart based on Pettitt (1976) test statistic for simultaneously monitoring the location and the scale parameters of any continuous process. The test statistic is a combination of Wilcoxon and Mood statistics. The single plotting statistic for the joint monitoring of location and scale is given T in equation (1) and chart is called Shewhart-Pettitt (SP) chart. We considered, $X = (X_1, X_2, ..., X_m)$, as reference sample of size *m* from an in-control process and that $Y = (Y_1, Y_2, ..., Y_n)$ an arbitrary test sample of size *n*.

Proposed Charting Procedure

Charting procedure of the proposed SP chart is same as that of the SL chart by Mukherjee and Chakraborti (2012). Only plotting statistic S^2 is replaced by the statistic T. The charting procedure of the SP chart is as follows:

Step 1. Collect a reference sample $X = (X_1, X_2, ..., X_m)$ from an in-control process.

Step 2. Assuming a subgroup size of *n* observations and denoting Y_i as the i^{th} sample, $i = 1, 2, \ldots$ compute the WRS statistic and Mood statistic for Y_i against the reference sample X and denote them as W_i and M_i respectively.

Step 3. Compute the means and standard deviations of WRS statistic and that of the Mood statistic respectively.

Step 4. Calculate the standardized WRS and Mood statistics $T_{1i} = \dfrac{(W_i - \mu_1)}{\sigma_1}$ and $T_{2i} = \dfrac{(M_i - \mu_2)}{\sigma_2}$ respectively.

Step 5. Calculate the Pettit plotting statistic $T_i = T_{1i}^2 + T_{2i}^2$, $i = 1, 2, \ldots.$

Step 6. Plot T_i against an upper control limit (UCL), $H > 0$. The lower control limit (LCL) is zero. Note that $T_i \geq 0$ and larger value of T_i suggest an out-of-control process.

Step 7. If T_i exceeds *H*, the process is declared out-of-control at the i^{th} test sample. If not, the process is thought to be in-control, and testing constitutes to the next test sample.

4. RUN LENGTH DISTRIBUTION AND ARL

In order to implement the proposed SP control chart, we need, H the upper control limit. This requires the run length distribution. Let RL be the random variable denoting the run length of the proposed SP chart. It is clear that, given the reference sample $X = (X_1, X_2, ..., X_m)$ the conditional run length distribution is geometric with probability of success $\Pr(T_i > H \mid X)$, where probability of success is the out-of-control signal. Therefore, all moments, percentiles etc. of the conditional distribution can be obtained directly from the properties of geometric distribution. The conditional ARL is given by $\dfrac{1}{\Pr(T_i > H \mid X)}$. Hence, all the properties of the unconditional run length distribution can be obtained by averaging over the distribution of reference sample. The unconditional ARL can be found from $E\left[\dfrac{1}{\Pr(T_i > H \mid X)}\right]$. Both the conditional and unconditional run length distributions can provide useful information about the performance of the chart.

Thus by writing $\psi(\mathrm{X},\mathrm{H}) = \Pr(\mathrm{T_i} < \mathrm{H} \mid \mathrm{X})$ for $i = 1, 2, \ldots$ the unconditional run length distribution is given by

$$\begin{aligned}\Pr(\mathrm{RL} = t) &= \mathrm{E}\left\{[\psi(\mathrm{X},\mathrm{H})]^{t-1}[1-\psi(\mathrm{X},\mathrm{H})]\right\} \\ &= \mathrm{E}[\psi(\mathrm{X},\mathrm{H})]^{t-1} - \mathrm{E}[\psi(\mathrm{X},\mathrm{H})]^{t}, \quad t = 1, 2, \ldots\end{aligned}$$

Moments of the unconditional run length distribution can be calculated by conditioning on the reference sample. The ARL is given by

$$\begin{aligned}ARL &= E\left[\frac{1}{1-\psi(\mathrm{X},\mathrm{H})}\right] \\ &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{1-\psi(\mathrm{X},\mathrm{H})}\, dF(X_1)\ldots dF(X_m) \qquad (2)\end{aligned}$$

The in-control ARL (ARL_0) can be obtained from equation (2) substituting $\mathrm{F} = \mathrm{G}$

$$\begin{aligned}ARL_0 &= E\left[\frac{1}{1-\psi_{F_1=F_2}(\mathrm{X},\mathrm{H})}\right] \\ &= \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{1-\psi_{F_1=F_2}(\mathrm{X},\mathrm{H})}\, dF(X_1)\ldots dF(X_m) \qquad (3)\end{aligned}$$

Other moments and percentiles of the run length distribution can be calculated in a similar manner. Using the analytical form of in-control ARL in equation (3), we can set ARL_0 equal to some desired (nominal) value, say, ARL_0^* and determine the constant H by solving

$$ARL_0^* = \int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\frac{1}{1-\psi_{F=G}(\mathrm{X},\mathrm{H})}\, dF(X_1)\ldots dF(X_m) \qquad (4)$$

Since the equation (4) is an *m* dimensional integration and $\psi(\mathrm{X},\mathrm{H})$ is generally not in closed form, a direct numerical solution to Equation (4) is not easy to obtain. Alternatively, Monte-Carlo simulation approach, covering sufficiently large number of possible samples can be used to approximate equation (4) and we used this approach here.

Since the chart is distribution-free, for the Phase-I sample, we generate *m* samples from the standard normal distribution and for each test sample, and we generate *n* observations from the same distribution. The computer programs written in C language are used to estimate ARL_0 and other percentiles of the distribution using 10000 replicates. We choose m=30 for the reference sample size and n=5for the test sample size. To find H for any given pair of (m,n) values, a search is conducted with different values of H, and that value of H is obtained for which ARL_0 is equal to a nominal (target) value. Table.1 presents the H (UCL) values for the target ARL_0 = 500 along with reference sample size m = 30 and test sample size 5.

Table 1. UCL of charting constants of the SP control chart for various values of *m* and n, and for some standard (target) values of in-control ARL = 500.

m	30	30	30	50	50	50	100	100	100
n	5	10	25	5	10	25	5	10	25
ARL=500	7.31	8.87	10.51	15.26	14.26	12.14	12.09	12.16	12.54

5. PERFORMANCE ANALYSIS OF THE SP CHART

In order to investigate the out-of-control performance of the proposed SP chart, we consider the underlying process distributions as normal, double exponential and uniform with mean zero and variance one. The uniform distribution is considered as process distribution to study the effect of a light tailed distribution and double exponential distribution is considered to study the effect of heavy tailed distribution on the performance of the SP chart. The distribution of observation from the process is considered to have mean zero and variance one for all process distributions under study.

Performance Analysis under Normal Distribution

In order to investigate the out-of-control performance of the proposed SP chart, we consider the underlying process distributions as normal, samples are taken from $\mathrm{N}(\theta,\lambda)$ distribution, with in-control sample coming from a $\mathrm{N}(0,1)$ distribution. To examine the effects of shifts in the mean and variance, 30 combinations of (θ,λ) values are considered with $\theta =$ *0, 0.25, 0.5, 1.0, 1.5* and $\lambda =$ *1.0, 1.25, 1.5, 1.75* and *2.0*. Table 2 present the performance characteristics of the SP chart when underlying process distribution is normal with the various combinations of the reference and test sample sizes *m* = 30 and *n* = 5.

Performance Analysis under Heavy Tailed Distribution

As we know the nonparametric control charts are in-control robust, that is, their in-control performance remains the same for all continuous distributions. Now we are interested to study the effect of heavy tailed underlying distribution on the performance of the proposed SP chart. Heavy tailed symmetric distributions such as double exponential distribution often arise in many applications. We conduct the simulation study with data from double exponential distribution. The performance characteristics of the run length distribution were evaluated when the in-control sample is from double exponential distribution with mean 0 and variance 1, and test samples are generated from double exponential distribution with mean θ and standard deviation λ. To examine the effects of shifts in location and scale, as in the normal case, we studied 30 combinations of (θ,λ) values. Table 3 present the performance characteristics of the proposed SP chart when underlying process distribution is double exponential with the various combinations of the reference and test sample sizes *m* = 30 and *n* = 5. It is observed that when underlying process distribution is double exponential distribution, the general patterns remain same the same as in the case of the normal process distribution. However, the out-of-control ARL values for detecting a shift in the mean or in the variance under double exponential process distribution are larger than that of the ARL values under normal process distribution. It indicates that the proposed SP chart detects shifts in process location and scale slightly slower under heavy tailed distribution. Moreover, the percentiles as well as the SDRL all increase under the double exponential distribution.

Table 2. Performance characteristics of the SP chart for normal distribution.

(ARL_0 =500, *m* = 30 and *n* = 5)

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1	500.87	507.60	25	145	346	695	1497

0.5	1	79.74	77.97	5	24	56	210	239
1.0	1	8.94	6.34	1	3	6	12	26
1.5	1	2.31	1.74	1	1	2	3	6
2.0	1	1.24	0.55	1	1	1	1	2
0.0	1.25	69.34	69.39	4	21	48	95	210
0.5	1.25	29.96	28.94	2	9	21	41	88
1.0	1.25	6.93	6.41	1	2	5	9	20
1.5	1.25	2.52	196	1	1	2	3	6
2.0	1.25	1.40	0.75	1	1	1	2	3
0.0	1.5	21.57	19.98	2	7	15	30	64
0.5	1.5	14.24	12.35	1	4	10	20	41
1.0	1.5	5.46	4.93	1	2	4	7	5
1.5	1.5	2.55	1.99	1	1	2	3	7
2.0	1.5	1.57	0.95	1	1	1	2	3
0.0	1.75	10.49	9.98	1	3	7	14	30
0.5	1.75	8.20	7.68	1	3	6	11	23
1.0	1.75	4.49	3.96	1	2	3	6	12
1.5	1.75	2.53	1.97	1	1	2	3	6
2.0	1.75	1.65	1.04	1	1	1	2	4
0.0	2.0	6.32	5.80	1	2	5	8	18
0.5	2.0	5.47	4.94	1	2	4	7	16
1.0	2.0	3.71	3.17	1	1	3	5	10
1.5	2.0	2.41	1.84	1	1	2	3	6
2.0	2.0	1.69	1.08	1	1	1	2	4

Table 3. Performance characteristics of the SP chart for double exponential distribution
(ARL_0 =500, m = 30 and n = 5).

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1	500.89	504.75	26	142	348	687	1518
0.5	1	289.31	291.27	15	84	203	394	876
1.0	1	72.48	71.14	4	21	50	100	214
1.5	1	18.53	16.80	1	6	13	25	55
2.0	1	6.02	5.50	1	2	4	8	17
0.0	1.25	119.30	118.97	7	35	82	166	361
0.5	1.25	86.99	86.50	5	26	60	121	262
1.0	1.25	34.34	33.23	2	10	24	47	101
1.5	1.25	12.60	10.65	1	4	9	17	36
2.0	1.25	5.29	4.76	1	2	4	7	15
0.0	1.5	44.39	43.61	3	13	31	61	134
0.5	1.5	37.47	36.00	2	11	26	52	111
1.0	1.5	19.98	18.74	1	6	14	28	60
1.5	1.5	9.40	6.96	1	3	7	13	27
2.0	1.5	4.74	4.21	1	2	3	6	13
0.0	1.75	21.64	20.24	2	7	15	30	63
0.5	1.75	19.45	17.94	1	6	14	27	57
1.0	1.75	12.76	10.58	1	4	9	18	37
1.5	1.75	7.32	6.80	1	2	5	10	21
2.0	1.75	4.22	3.69	1	2	3	6	12
0.0	2.0	12.88	11.01	1	4	9	17	37

0.5	2.0	12.08	9.83	1	4	9	17	35
1.0	2.0	8.75	6.11	1	3	6	12	25
1.5	2.0	5.76	5.24	1	2	4	8	16
2.0	2.0	3.75	3.21	1	1	3	5	10

To study the effect of light tailed underlying distribution on the performance of the proposed SP chart, uniform distribution is included in the study as process distribution. We conduct the simulation study with data from uniform distribution. The performance characteristics of the run length distribution were evaluated when the in-control sample is from uniform distribution with mean 0 and variance 1, and test samples are generated from uniform distribution with mean θ and standard deviation λ. To examine the effects of shifts in location and scale, as in the normal case, we studied 30 combinations of (θ,λ) values.

Table 4, it is observed that when underlying process distribution is uniform, the general patterns remain the same as in the case of the normal process distribution. However, the out-of-control ARL values for detecting a shift in the mean or in the variance under uniform process distribution are smaller than that of the ARL values under normal process distribution.

Table 4. Performance characteristics of SP chart for Uniform distribution.

(ARL_0 =500, m = 30 and n =5)

θ	λ	ARL	SDRL	P_5	Q_1	Q_2	Q_3	P_{95}
0.0	1	505.97	505.95	26	145	350	705	1506
0.5	1	64.01	62.37	4	19	45	90	190
1.0	1	10.74	8.56	1	3	3	15	31
1.5	1	3.49	2.95	1	1	3	5	10
2.0	1	1.69	1.08	1	1	1	2	4
0.0	1.25	28.65	27.02	2	9	20	40	83
0.5	1.25	21.19	20.06	1	6	15	29	63
1.0	1.25	8.69	6.07	1	3	6	12	25
1.5	1.25	3.79	3.25	1	1	3	5	10
2.0	1.25	2.06	1.48	1	1	2	3	5
0.0	1.5	9.77	9.26	1	3	7	13	28
0.5	1.5	8.75	8.23	1	3	6	12	25
1.0	1.5	6.68	6.16	1	2	5	9	19
1.5	1.5	3.97	3.43	1	1	3	5	11
2.0	1.5	2.37	1.80	1	1	2	3	6
0.0	1.75	5.50	4.97	1	2	4	7	16
0.5	1.75	5.36	4.83	1	2	4	7	15
1.0	1.75	4.48	3.95	1	2	3	6	12
1.5	1.75	3.80	3.26	1	1	3	5	10
2.0	1.75	2.60	2.04	1	1	2	3	7
0.0	2.0	3.78	3.24	1	1	3	5	10
0.5	2.0	3.78	3.24	1	1	3	5	10
1.0	2.0	3.47	2.93	1	1	3	5	9
1.5	2.0	3.06	2.51	1	1	2	4	8
2.0	2.0	2.77	2.21	1	1	2	4	7

6. CONCLUSIONS

In this chapter, a single nonparametric control chart, called the SP chart is developed for joint monitoring of location and scale parameters of a continuous process distribution. Both in-control and out-of-control performance of the chart are studied under normal, heavy tailed double exponential and light tailed uniform process distributions. It is observed that the proposed SP chart maintain its designed in-control ARL under the considered process distributions. The chart is found to be more efficient under light tailed distribution, where as it is less efficient under heavy tailed distribution as compared to normal distribution. The various performance characteristics such as mean, median and some percentiles of the run length distribution are examined.

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